AS Level Further Mathematics

Topic: Cirular Motion

Teacher:

Muhammad Shafiq ur Rehman

Aitchison College Lahore

Q.01 Oct/Nov/P31/2022

A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute. [3]

Given $m = 2\text{ kg}$, $r = 0.6\text{ m}$, $T = 20\text{ N}$

use $F = ma$

$T = m\frac{v^2}{r}$ as $a = \frac{v^2}{r}$ or $a = r\omega^2$

$20 = 2\frac{v^2}{0.6}$

$v^2 = 6$

$v = \sqrt{6}$

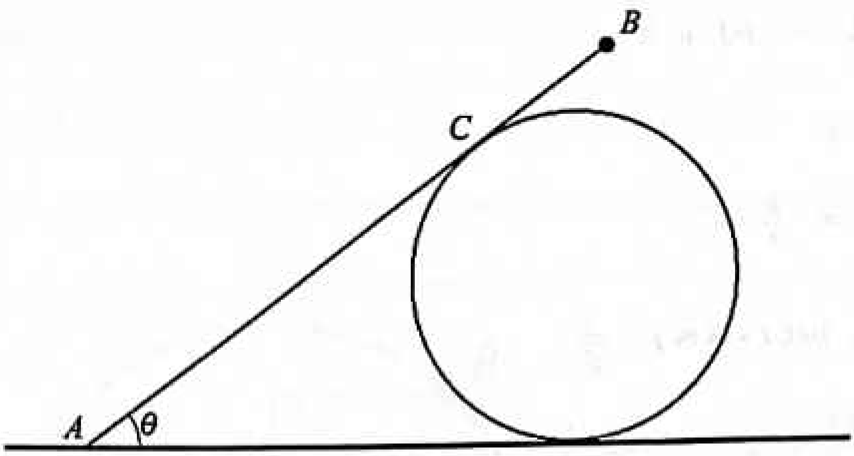
No of revolution per min $= \frac{60v}{2\pi r}$

$= \frac{60\sqrt{6}}{2\pi(0.6)}$

$= 38.98 = 39$

Q.2 Oct/Nov/P31+P33/2022

3

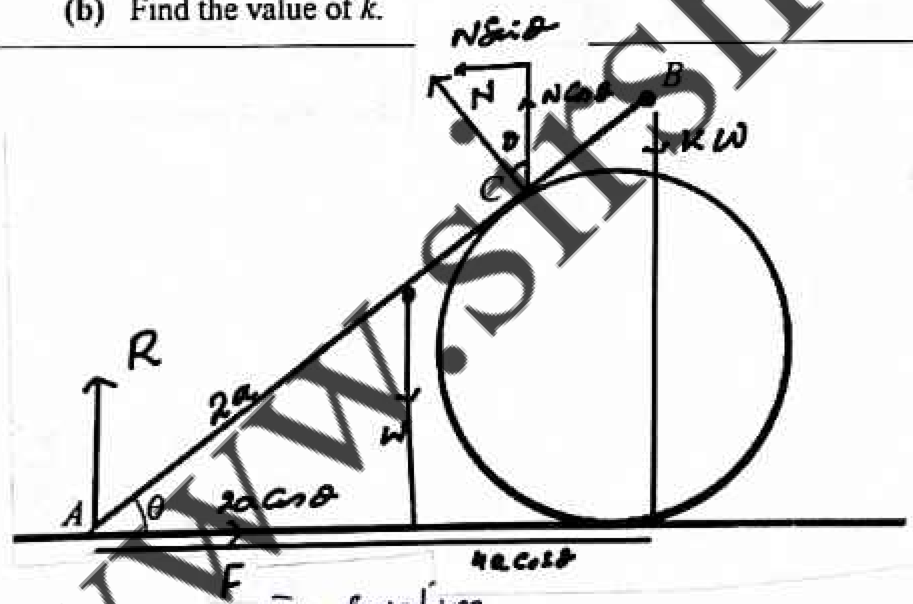


A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod AB, of length $4a$ and weight W, rests against the surface of the cylinder. The end A of the rod is in contact with the horizontal surface. The vertical plane containing the rod AB is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is C, where $AC = 3a$. The angle between the rod and the horizontal surface is θ where $\tan\theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight kW is attached to the rod at B. The rod is about to slip. The normal reaction between the rod and the cylinder is N.

(a) Show that $N = \frac{8}{15}W(1 + 2k)$. [2]

(b) Find the value of k. [5]



Let F = be the friction

R = Normal reaction at A

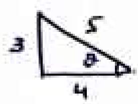
Take moments about A

$N \times 3a = W \times 2a\cos\theta + kW \times 4a\cos\theta$

$3N = (2 + 4k)W\cos\theta$

$N = (2 + 4k)W \cdot \frac{4}{5} \cdot \frac{1}{3} = \frac{8}{15}W(1 + 2k)$

Given $\tan\theta = \frac{3}{4}$



so $\cos\theta = \frac{4}{5}$

Resolve the forces vertically and equate

$\uparrow \quad N\cos\theta + R = W + KW$

$\rightarrow \quad F = N\sin\theta$

$F = \mu R = \frac{6}{7}R$

$\frac{6}{7}R = \frac{8}{15}W(1+2K)\cdot\frac{3}{5}$

$R = \frac{28}{75}W(1+2k)$

Also $\quad R = \frac{21}{45}w(1+K)$

$\frac{28}{75}w(1+2K) = \frac{21}{45}w(1+K)$

$\frac{4}{5}(1+2K) = (1+K)$

$4+8K = 5+5K$

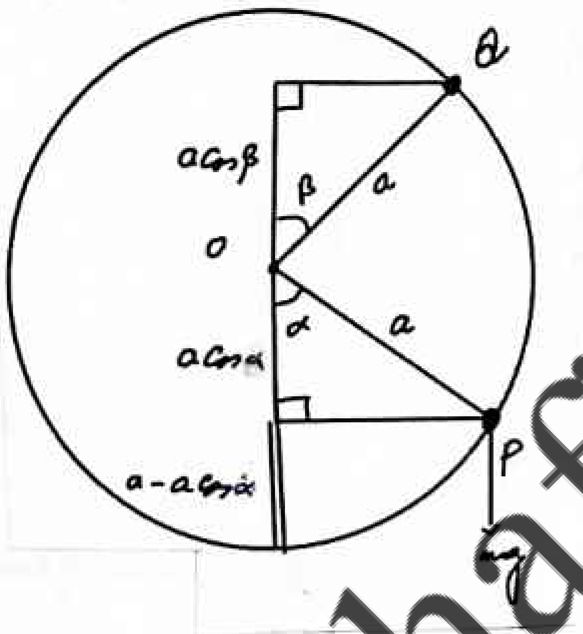
$3K = 1$

$K = \frac{1}{3}$

Q.1 Oct/Nov/P32/2022

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP making an angle α with the downward vertical, where $\cos\alpha = \frac{2}{3}$. The particle P is projected perpendicular to OP in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane.

Find the cosine of the angle between the string and the upward vertical when the string first becomes slack. [4]



Let the string goes slack at Q and v be the velocity of the particle at Q.

So at Q $T = 0$

use $F = ma$

$$T + mg\cos\beta = m\frac{v^2}{a}$$

$$0 + mg\cos\beta = m\frac{v^2}{a}$$

$$v^2 = ag\cos\beta$$

Energy at P $= \frac{1}{2}mu^2 + mg(a - a\cos\alpha)$

at Q $= \frac{1}{2}mv^2 + mg(a + a\cos\beta)$

$$\frac{1}{2}mu^2 + mg(a - a\cos\alpha) = \frac{1}{2}mv^2 + mg(a + a\cos\beta)$$

$$u^2 + 2ag - 2ag\cos\alpha = v^2 + 2ag + 2ag\cos\beta$$

→

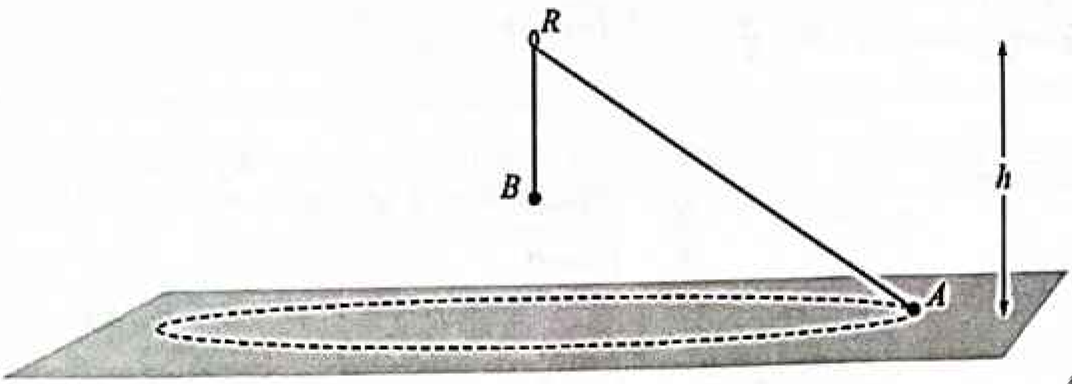
$$3ag - 2a\left(\frac{2}{3}\right)g = ag\cos\beta + 2ag\cos\beta$$

$$\frac{5}{3} = 3\cos\beta$$

$$\cos\beta = \frac{5}{9}$$

Q.4 Oct/Nov/P32/2022

6



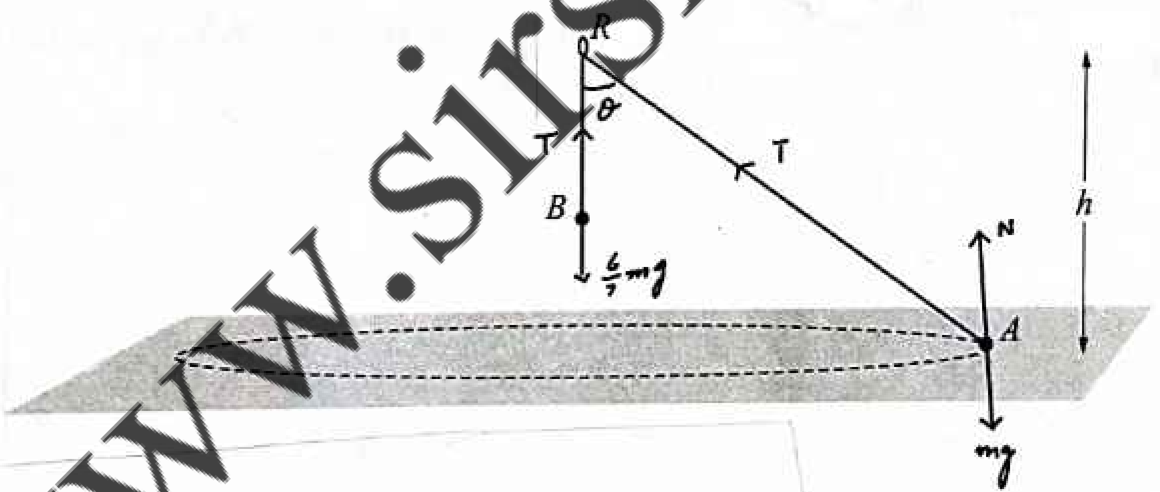
A light inextensible string is threaded through a fixed smooth ring R which is at a height h above a smooth horizontal surface. One end of the string is attached to a particle A of mass m. The other end of the string is attached to a particle B of mass $\frac{6}{7}m$. The particle A moves in a horizontal circle on the surface. The particle B hangs in equilibrium below the ring and above the surface (see diagram).

When A has constant angular speed ω, the angle between AR and BR is θ and the normal reaction between A and the surface is N.

When A has constant angular speed $\frac{3}{2}\omega$, the angle between AR and BR is α and the normal reaction between A and the surface is $\frac{1}{2}N$.

(a) Show that $\cos\theta = \frac{4}{9}\cos\alpha$. [5]

(b) Find N in terms of m and g and find the value of $\cos\alpha$. [4]

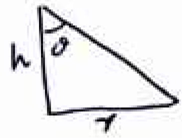


(a) $T = \frac{6}{7}mg$

use $F = ma$ when angular speed $= \omega$ and angle θ

$T\sin\theta = m\omega^2 r$

$\frac{6}{7}mg\sin\theta = m\omega^2 h\tan\theta$ — ① as



$\frac{r}{h} = \tan\theta$

$r = h\tan\theta$

when angular speed $= \frac{3}{2}\omega$

and angle $= \alpha$, then

$$T \sin\alpha = m\left(\tfrac{3}{2}\omega\right)^2 r$$

$$\tfrac{6}{7} mg \sin\alpha = m \tfrac{9}{4}\omega^2 h \tan\alpha \quad \text{— (2)}$$

By (1) ÷ (2)

$$\frac{\sin\theta}{\sin\alpha} = \frac{4}{9}\,\frac{\tan\theta}{\tan\alpha}$$

$$\frac{\sin\theta}{\tan\theta} = \frac{4}{9}\,\frac{\sin\alpha}{\tan\alpha}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{9}\cos\alpha$$

(b)

$$T\cos\theta + N = mg \quad \text{— (3)}$$

$$T\cos\alpha + \tfrac{1}{2}N = mg \quad \text{— (4)}$$

$$T\left(\tfrac{4}{9}\cos\alpha\right) + N = mg \qquad \text{as } \cos\theta = \tfrac{4}{9}\cos\alpha$$

$$T\cos\alpha = \tfrac{9}{4}(mg - N)$$

putting the value of $T\cos\alpha$ in (4)

$$\tfrac{9}{4}(mg - N) + \tfrac{1}{2}N = mg$$

$$-\tfrac{7}{4}N = -\tfrac{5}{4}mg$$

$$N = \tfrac{5}{7}mg$$

Q.5 May/June/P31+P32/2022

2 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string. The particle is held at the point A with the string taut. The angle between OA and the downward vertical is equal to α, where $\cos\alpha = \frac{4}{5}$. The particle is projected from A, perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle θ with the upward vertical through O.

Find the value of $\cos\theta$. [5]

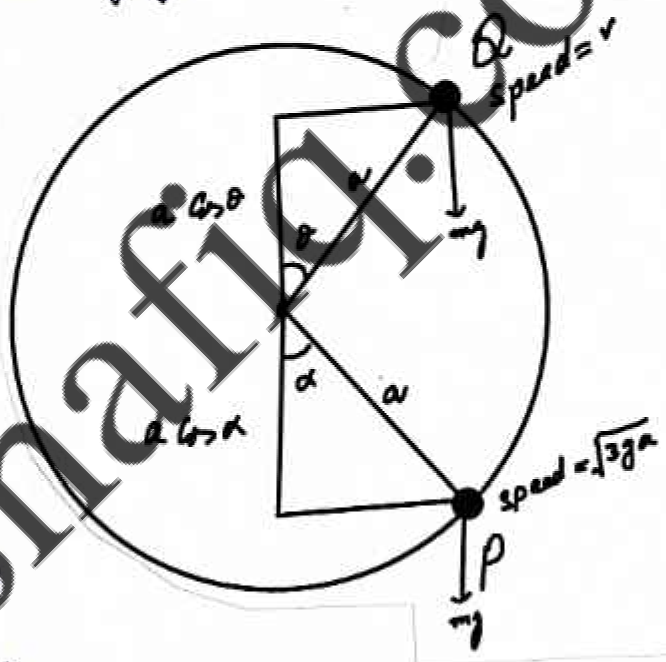
Let v be the speed at Q when string goes slack

At Q $T = 0$

use $F = ma$

$$mg\cos\theta = m\frac{v^2}{a}$$

$$v^2 = ag\cos\theta$$



By Energy equation

KE at P + PE at P = KE at Q + PE at Q

$$\frac{1}{2}m(\sqrt{3ga})^2 + mg(a - a\cos\alpha) = \frac{1}{2}mv^2 + mg(a + a\cos\theta)$$

$$\frac{3}{2}mga + mga - mga\left(\frac{4}{5}\right) = \frac{1}{2}mga\cos\theta + mga + mga\cos\theta$$

$$\frac{3}{2} - \frac{4}{5} = \frac{1}{2}\cos\theta + \cos\theta$$

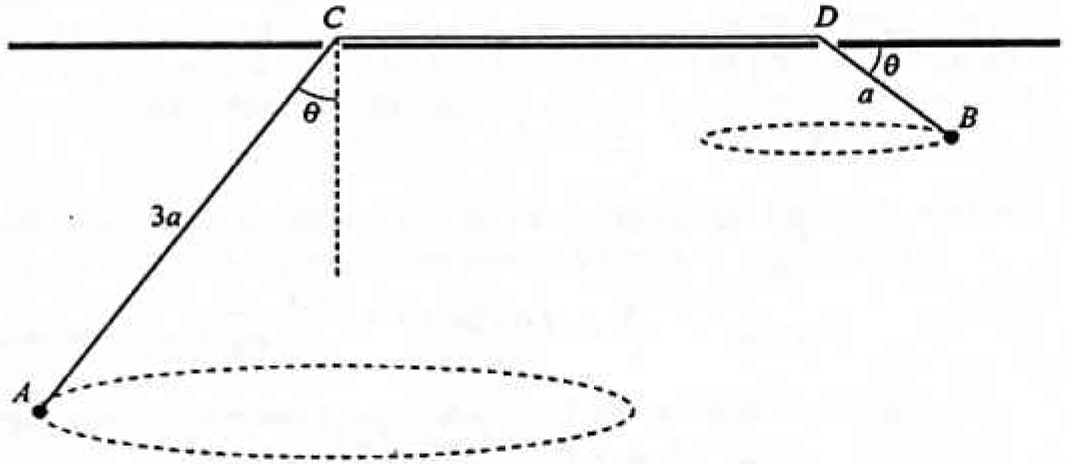
$$\frac{7}{10} = \frac{3}{2}\cos\theta$$

$$\cos\theta = \frac{7}{10} \times \frac{2}{3}$$

$$\cos\theta = \frac{7}{15}$$

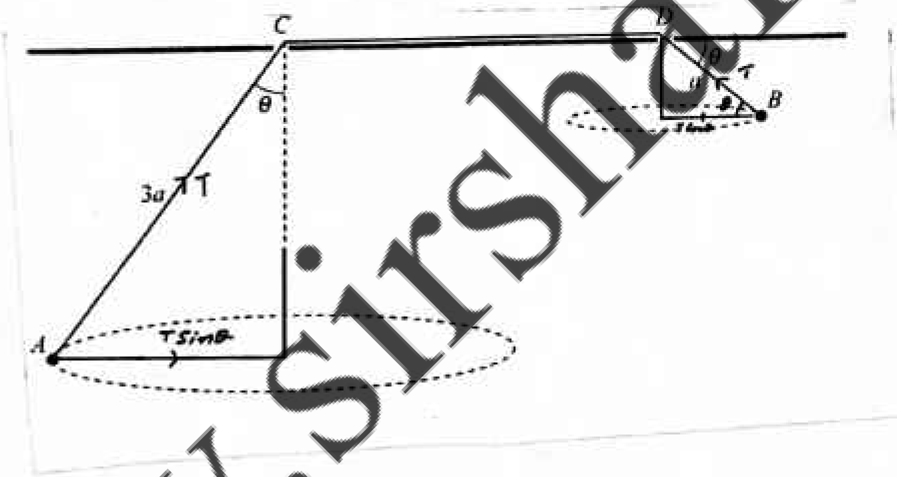
Q.2 May/June/P31+P32/2022

5



A light inextensible string AB passes through two small holes C and D in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass m is attached at the end A and moves in a horizontal circle with angular velocity ω. A particle of mass $\frac{3}{4}m$ is attached to the end B and moves in a horizontal circle with angular velocity $k\omega$. AC makes an angle θ with the downward vertical and DB makes an angle θ with the horizontal (see diagram).

Find the value of k. [7]



Let T be the tension in the string

For the particle A mass $= m$, angular velocity $= \omega$

use $F = ma$

$T \sin\theta = m r \omega^2$

$r = 3a \sin\theta$

$T \sin\theta = 3a\, m \sin\theta\, \omega^2$

$T = 3am\omega^2$ — ①

For the particle B   mass $= \frac{3}{4}m$, Angular velocity $= K\omega$

use $F = ma$

$$T\cos\theta = \frac{3}{4} m r (K\omega)^2$$

$$r = a\cos\theta$$

$$T\cos\theta = \frac{3}{4} m (a\cos\theta) K^2\omega^2$$

$$T = \frac{3}{4} m a K^2\omega^2 \quad \text{— (2)}$$

From (1) and (2)

$$3ma\omega^2 = \frac{3}{4} m a K^2\omega^2$$

$$K^2 = 4$$

$$\boxed{K = 2}$$

Q.3 May/June/P33/2022

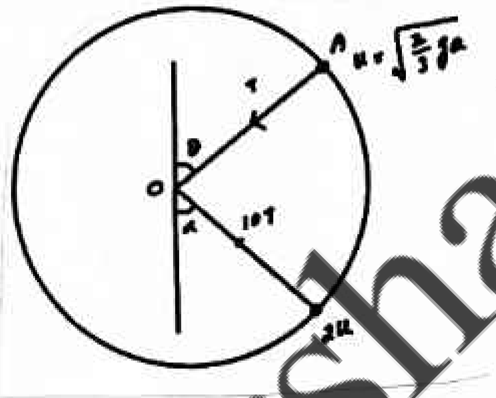
4 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string and is held with the string taut at the point A. At A the string makes an angle θ with the upward vertical through O. The particle is projected perpendicular to the string in a downward direction from A with a speed u. It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O, the speed of the particle is $2u$ and the magnitude of the tension in the string is 10 times its magnitude at A.

It is given that $u = \sqrt{\frac{2}{3}ga}$.

(a) Find, in terms of m and g, the magnitude of the tension in the string at A. [6]

(b) Find the value of $\cos\alpha$. [2]



(a) At A use F = ma

$$T + mg\cos\theta = m\frac{u^2}{a} \quad \text{—— (1)}$$

At B $$10T - mg\cos\alpha = m\frac{(2u)^2}{a} \quad \text{—— (2)}$$

Also Energy at A = Energy at B

$$\frac{1}{2}mu^2 + mg(a + a\cos\theta) = \frac{1}{2}m(2u)^2 + mg(a - a\cos\alpha)$$

$$u^2 + 2ag + 2ag\cos\theta = 4u^2 + 2ag - 2ag\cos\alpha$$

$$3u^2 = 2ag(\cos\theta + \cos\alpha)$$

$$3\cdot\frac{2}{3}ga = 2ag(\cos\theta + \cos\alpha)$$

→

$\cos\theta + \cos\alpha = 1$

From ① and ②

① − ②

$$-9T + mg(\cos\theta + \cos\alpha) = \frac{m}{a} \cdot \frac{2}{3} ga - \frac{m}{a} \times 4 \times \frac{2}{3} ga$$

$$-9T + mg(1) = \frac{2}{3} mg - \frac{8}{3} mg$$

$$-9T = -3mg$$

$$\boxed{T = \frac{1}{3} mg}$$

(b) putting the value of T in ②

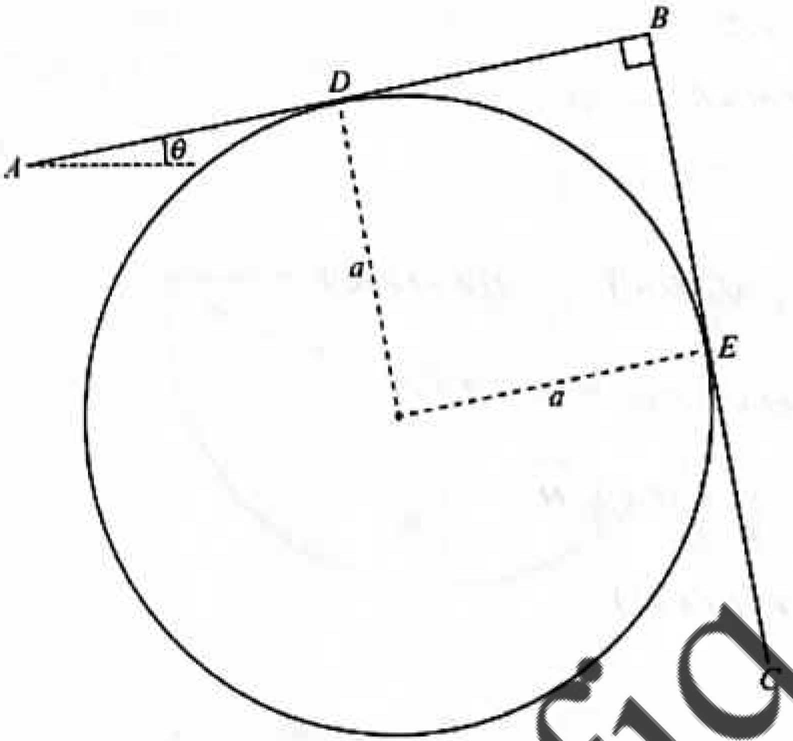
$$10 \times \frac{1}{3} mg - mg\cos\alpha = \frac{8}{3} mg$$

$$\frac{10}{3} - \frac{8}{3} = \cos\alpha$$

$$\boxed{\cos\alpha = \frac{2}{3}}$$

Q.8 May/June/P33/2022

7



A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. **[6]**

(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. **[3]**

μR = Frictional force at D

μN = Frictional force at E

Moments about the centre of the cylinder

$(W)(a\cos\theta) = (\mu N)(a) + (\mu R)(a) + W(a\sin\theta)$

$Wa(\cos\theta - \sin\theta) = a\mu(N+R)$

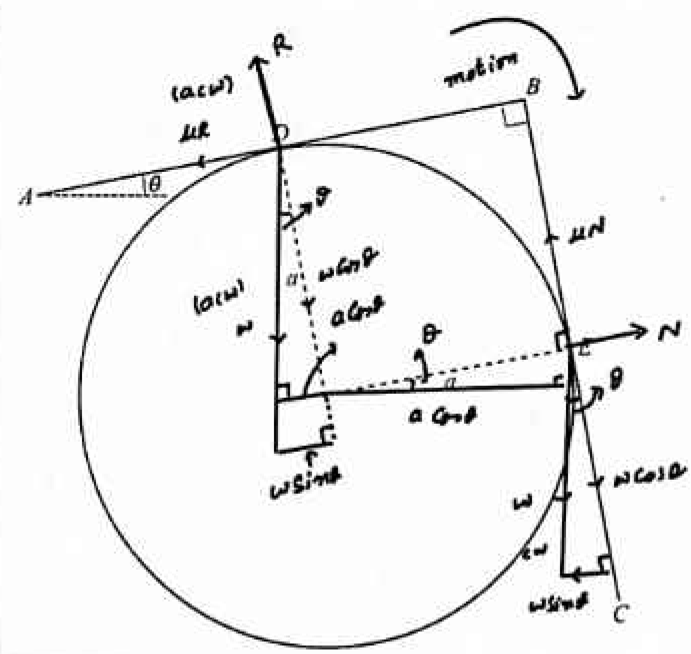
$W(\cos\theta - \sin\theta) = \mu(N+R)$ — ①

~~Resull Resultant~~ Resultant force along $AB = 0$

$N - W\sin\theta - W\sin\theta - \mu R = 0$

$2W\sin\theta = N - \mu R$

$W\sin\theta = \frac{1}{2}(N - \mu R)$ — ②



Resultant force along CB = 0

$\mu N + R - w\cos\theta - w\cos\theta = 0$

$2w\cos\theta = \mu N + R$

$w\cos\theta = \frac{1}{2}(\mu N + R)$ — (3)

Substitute (2) and (3) in (1)

$\frac{1}{2}(\mu N + R) - \frac{1}{2}(N - \mu R) = \mu(N + R)$

$\frac{1}{2}\mu N + \frac{1}{2}R - \frac{1}{2}N + \frac{1}{2}\mu R = \mu N + \mu R$

$\frac{1}{2}R - \frac{1}{2}\mu R = \frac{1}{2}\mu N + \frac{1}{2}N$

$R(1-\mu) = N(1+\mu)$

$\frac{R}{N} = \frac{1+\mu}{1-\mu}$

$R : N = 1+\mu : 1-\mu$

(b) Given $\mu = \frac{1}{3}$

$\frac{R}{N} = \frac{1 + 1/3}{1 - 1/3}$

$R = 2N$

(2) ÷ (3) $\tan\theta = \frac{N - \mu R}{\mu N + R}$

$\tan\theta = \frac{N - 2\mu N}{\mu N + 2N}$

$= \frac{N(1 - 2\mu)}{N(\mu + 2)}$

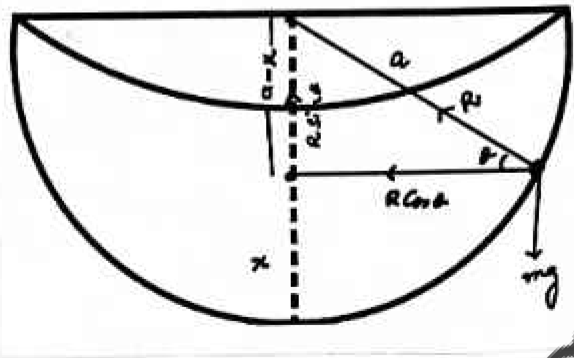
$= \frac{1 - \frac{2}{3}}{\frac{1}{3} + 2}$

$= \frac{\frac{1}{3}}{\frac{7}{3}} = \frac{1}{7}$

Q.4 May/June/P31+P32/2021

2 A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a. [6]



Let R be the normal reaction of the surface

$\uparrow \quad R\sin\theta = mg \quad —①$

$\rightarrow \quad R\cos\theta = m\dfrac{v^2}{r}$

$r = a\cos\theta$

$R\cos\theta = \dfrac{m\left(\sqrt{\frac{8}{3}ag}\right)^2}{a\cos\theta}$

$R\cos^2\theta = \dfrac{8}{3}mg$

$\dfrac{mg}{\sin\theta}\cos^2\theta = \dfrac{8}{3}mg$ from ①

$\dfrac{\cos^2\theta}{\sin\theta} = \dfrac{8}{3}$

$3(1-\sin^2\theta) = 8\sin\theta$

$3\sin^2\theta + 8\sin\theta - 3 = 0$

$\sin\theta = \frac{1}{3}$, $\sin\theta = -3$ (not possible)

$\dfrac{a-x}{a} = \sin\theta$

$a - x = a\sin\theta$

$x = (1-\sin\theta)a = (1-\frac{1}{3})a$

$x = \frac{2}{3}a$

Q.10 May/June/P31+P32/2021

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle completes vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O.

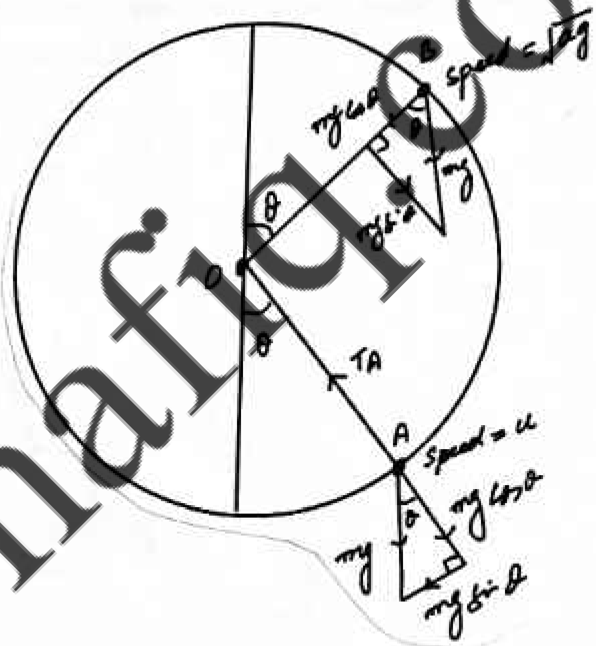
The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g. [8]

At A

use $F = ma$

$T_A - mg\cos\theta = m\frac{u^2}{a}$



At B

$T_B + mg\cos\theta = \frac{m(\sqrt{ag})^2}{a}$

$T_B + mg\cos\theta = mg$

Given $T_A = 7T_B$

$m\frac{u^2}{a} + mg\cos\theta = 7(mg - mg\cos\theta)$

$u^2 + ag\cos\theta = 7ag - 7ag\cos\theta$

$u^2 = ag(7 - 8\cos\theta)$ — ①

By Energy equation

KE at A + PE at A = KE at B + PE at B

$\frac{1}{2}mu^2 + mg(a - a\cos\theta) = \frac{1}{2}mag + mg(a + a\cos\theta)$

$u^2 + 2ag - 2ag\cos\theta = ag + 2ag + 2ag\cos\theta$

$u^2 = ag(4\cos\theta + 1)$ — ②

From ① and ②

$ag(7 - 8\cos\theta) = ag(4\cos\theta + 1)$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

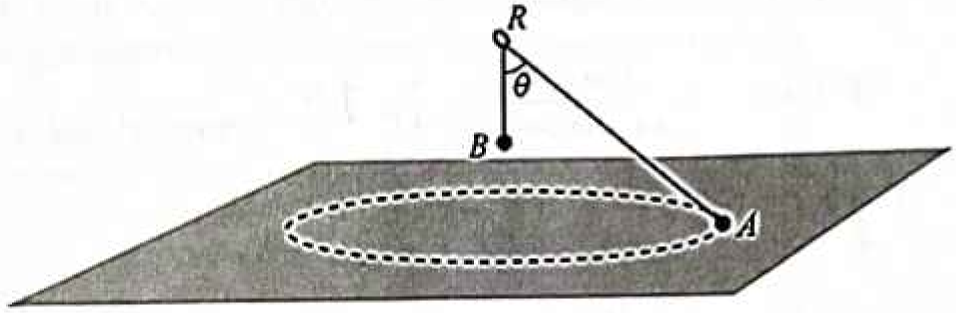
$u^2 = ag(7 - 8 \times \frac{1}{2})$

$u^2 = 3ag$

$u = \sqrt{3ag}$

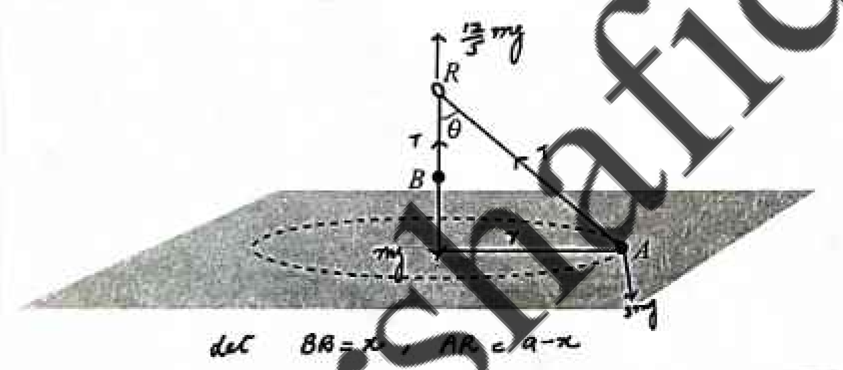
Q.5 May/June/P33/2021

3



Particles A and B, of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos\theta$. [3]



(a) Let T be the tension in the string

$$T = mg$$

$$R + T\cos\theta = 3mg \quad (R \text{ be the normal reaction of the ring})$$

$$\frac{12}{5}mg + mg\cos\theta = 3mg$$

$$\frac{12}{5} + \cos\theta = 3$$

$$\cos\theta = 3 - \frac{12}{5}$$

$$\cos\theta = \frac{3}{5}$$

(b) Let $BR = x$, $AR = a - x$

$$T\sin\theta = 3m\frac{v^2}{r}$$

$$r = (a-x)\sin\theta$$

$$T\sin\theta = \frac{3m}{(a-x)\sin\theta}\cdot\frac{4}{25}ga$$

$$mg(a-x)\sin^2\theta = \frac{12}{25}mga$$

$$(a-x)\cdot\frac{16}{25} = \frac{12}{25}a$$

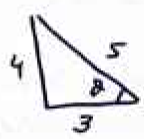
$$a-x = \frac{12}{16}a$$

$$a-x = \frac{3}{4}a$$

$$a-\frac{3}{4}a = x$$

$$x = \frac{1}{4}a$$

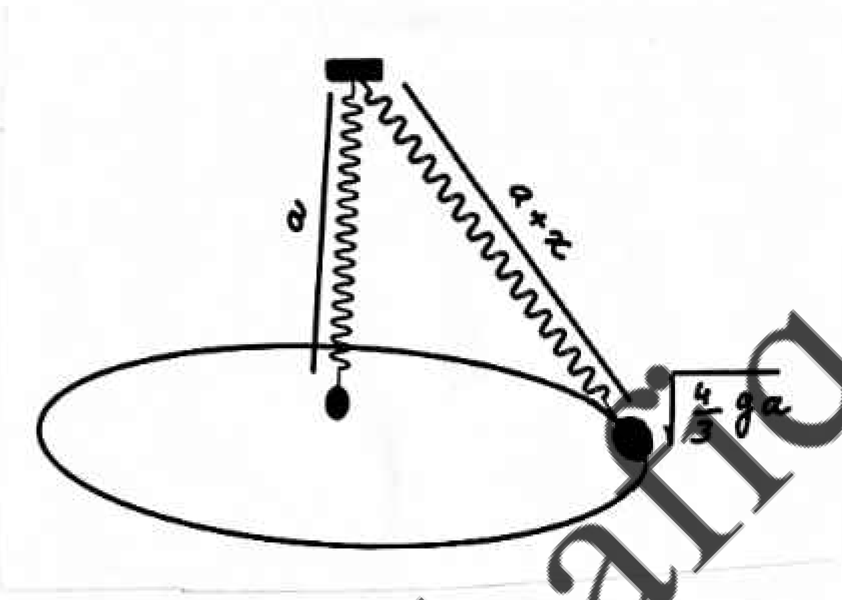
$\cos\theta = \frac{3}{5}$



Q.6 Oct/Nov/P31+P33/2021

1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]



Given $l = a$ $\lambda = 3mg$

let x = be the extension

By Hooks law

$$T = \frac{\lambda x}{l} = \frac{3mgx}{a} \quad \text{— ①}$$

use $F = ma$

$$T = m\frac{v^2}{r}$$

$$T = \frac{m\,\frac{4}{3}ga}{a+x} = \frac{4mga}{3(a+x)} \quad \text{— ②}$$

From ① and ②
$$\frac{3mgx}{a} = \frac{4mga}{3(a+x)}$$

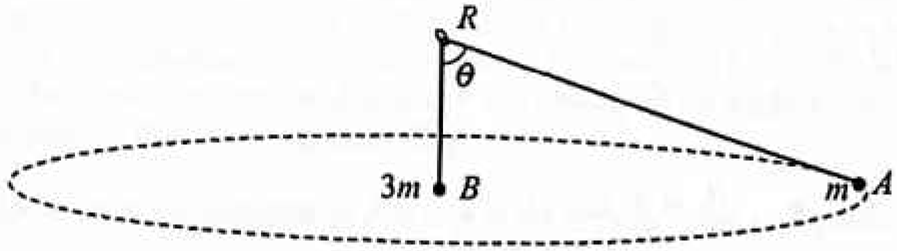
$$9x^2 + 9ax - 4a = 0$$

$$(3x - a)(3x + 4a) = 0$$

$x = \frac{a}{3}$ $x = -\frac{4a}{3}$ (not possible)

Q.13 Oct/Nov/P32/2021

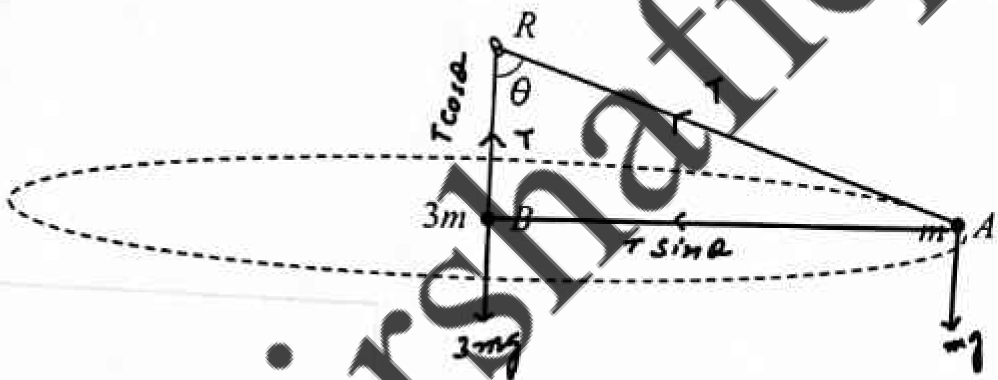
3



Particles A and B, of masses m and $3m$ respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles with speed v. Particles A and B are at the same horizontal level. The angle between AR and BR is θ (see diagram).

(a) Show that $\cos\theta = \frac{1}{3}$. [2]

(b) Find an expression for v in terms of a and g. [4]



From the diagram

$T = 3mg$ — ①

$T\cos\theta = mg$ — ②

$3mg\cos\theta = mg$

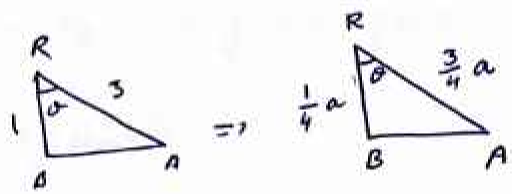
$\cos\theta = \frac{1}{3}$

(b) Let $|BR| = x$, then $|AR| = a - x$

use $F = ma$

$T\sin\theta = m\frac{v^2}{r}$

Since $\cos\theta = \frac{1}{3}$, so



$\sin\theta = \frac{AB}{AR} \Rightarrow r = \frac{3}{4}a\sin\theta$

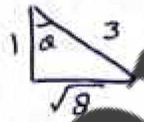
→

$$T\sin\theta = \frac{mv^2}{r}$$

$$3mg\sin\theta = \frac{mv^2}{\frac{3}{4}a\sin\theta}$$

$$v^2 = \frac{9}{4}ag\sin^2\theta$$

$$v^2 = \frac{9}{4}ag\cdot\left(\frac{\sqrt{8}}{3}\right)^2 \quad \text{as } \cos\theta = \frac{1}{3}$$



$$v^2 = 2ag$$

$$v = \sqrt{2ag}$$

Q.7 Oct/Nov/P32/2021

6 A particle P, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is 9 : 5.

(a) Find the value of $\cos\theta$. [6]

(b) Find, in terms of a and g, the greatest speed of P during its motion. [2]

(a) At A

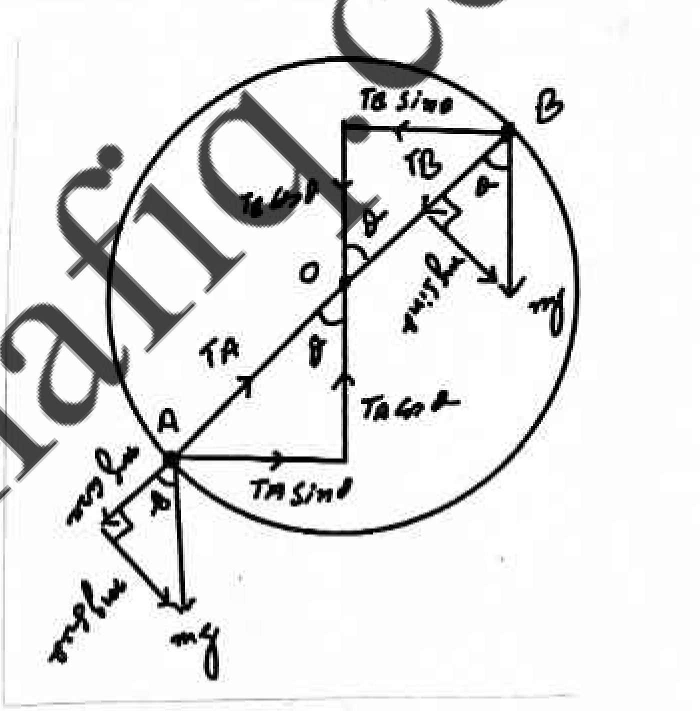
$$T_A - mg\cos\theta = m\frac{v_A^2}{r}$$

$$= m \cdot \frac{5ag}{a}$$

$$= 5mg$$

$$T_A = 5mg + mg\cos\theta$$

$$= mg(5+\cos\theta)$$



At B

$$T_B + mg\cos\theta = m\frac{v_B^2}{r}$$

$$= \frac{m v_B^2}{a}$$

According to Energy equation

K.E at A + P.E at A = K.E at B + P.E at B

$$\frac{1}{2}m(\sqrt{5ag})^2 + mg(a - a\cos\theta) = \frac{1}{2}mv_B^2 + mg(a + a\cos\theta)$$

$$5ag + 2ag - 2ag\cos\theta = v_B^2 + 2ag + 2ag\cos\theta$$

$$v_B^2 = 5ag - 4ag\cos\theta$$

so

$$T_B = \frac{m}{a}(5ag - 4ag\cos\theta) - mg\cos\theta$$

Given $T_A : T_B = 9:5$

$$\frac{mg(5+\cos\theta)}{mg(5-5\cos\theta)} = \frac{9}{5}$$

$$25+5\cos\theta = 45-45\cos\theta$$

$$50\cos\theta = 20$$

$$\cos\theta = \frac{2}{5}$$

(b) speed will be greatest at lowest point

let v be the greatest speed

$$\frac{1}{2}m(\sqrt{5ag})^2 + mg(a-a\cos\theta) = \frac{1}{2}mv^2$$

$$5ag + 2ga(1-\cos\theta) = v^2$$

$$v^2 = 5ag + 2ag\left(1-\frac{2}{5}\right)$$

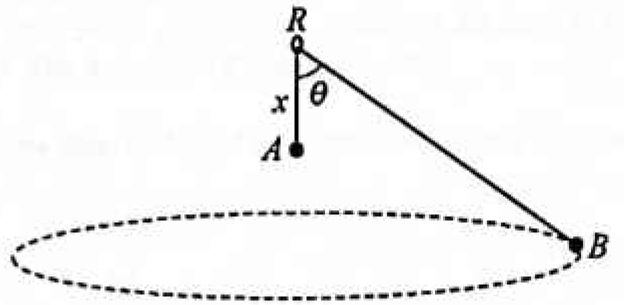
$$v^2 = 5ag + \frac{6}{5}ag$$

$$v^2 = \frac{31}{5}ag$$

$$v = \sqrt{\frac{31}{5}ag}$$

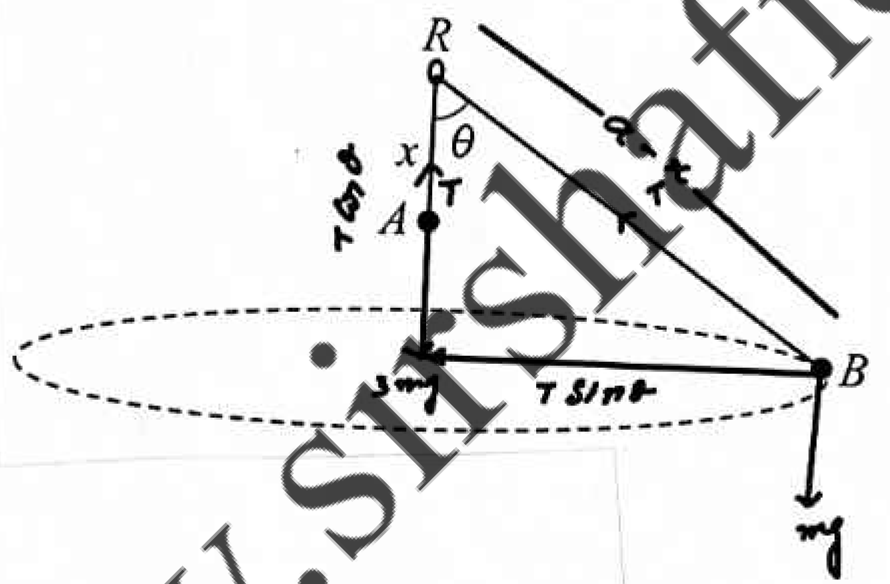
Q.8 May/June/P31+32/2020

2



A light inextensible string of length a is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass $3m$. The other end of the string is attached to a particle B of mass m. The particle A hangs in equilibrium at a distance x vertically below the ring. The angle between AR and BR is θ (see diagram). The particle B moves in a horizontal circle with constant angular speed $2\sqrt{\frac{g}{a}}$.

Show that $\cos\theta = \frac{1}{3}$ and find x in terms of a. [5]



$T = 3mg$ —— (1)

$T\cos\theta = mg$ —— (2)

From ① and ②

$3mg\cos\theta = mg$

$\cos\theta = \frac{1}{3}$

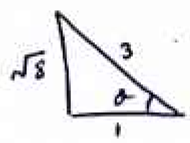
use $F = ma$

$T\sin\theta = m r\omega^2$

$3mg\sin\theta = m(a-x)\sin\theta \cdot \left(2\sqrt{\frac{g}{a}}\right)^2$

$3 = (a-x)\cdot\frac{4}{a} \Rightarrow 3a = 4a - 4x$

$x = \frac{1}{4}a$



$\sin\theta = \frac{\sqrt{8}}{3}$

Q.9 May/June/P33/2020

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O. The tension in the string is $4mg$.

Find, in terms of a and g, the time that P takes to make one complete revolution. [2]

Given $T = 4mg$ —(1)

Also we can write

$$T = ma$$

$$T = mr\omega^2 \text{ —(2)}$$

From (1) and (2)

$$ma\omega^2 = 4mg$$

$$\omega^2 = \frac{4g}{a}$$

Time for one complete revolution $= \frac{2\pi}{\omega}$

$$= \frac{2\pi}{2\sqrt{\frac{g}{a}}}$$

$$= \pi\sqrt{\frac{a}{g}}$$

Q.10 May/June/P33/2020

3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a. The particle moves in complete vertical circles about O. The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O. The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A : T_B$. [6]

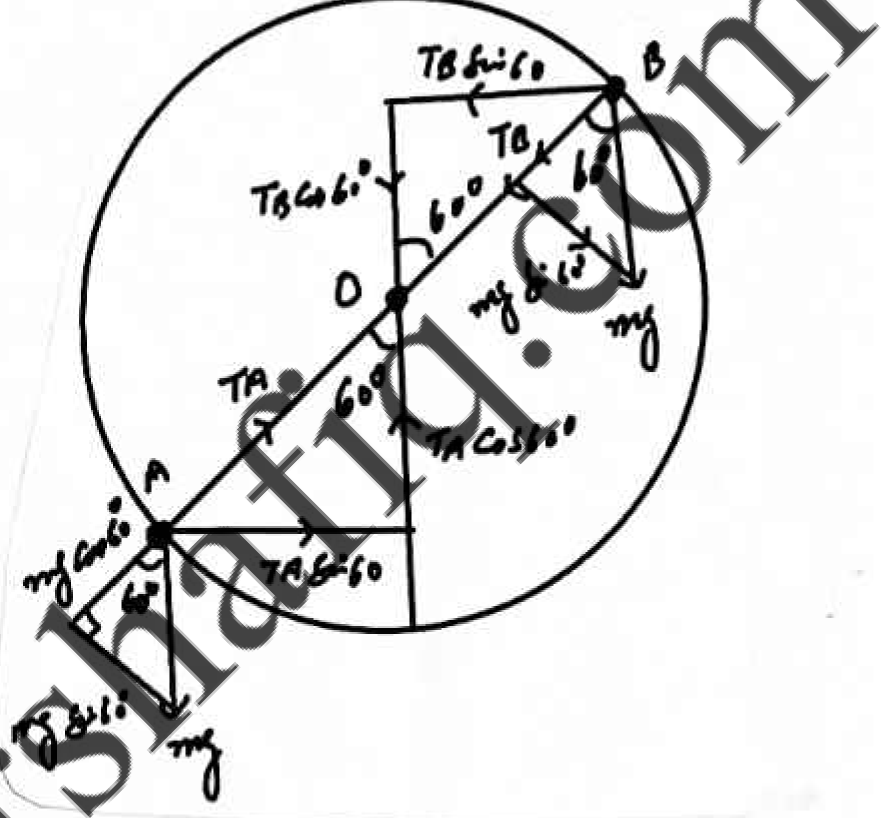
At A

$T_A - mg\cos 60 = m\dfrac{v_A^2}{r}$

$T_A = \dfrac{m(2\sqrt{ag})^2}{a} + \frac{1}{2}mg$

$T_A = 4mg + \frac{1}{2}mg$

$T_A = \frac{9}{2}mg$



At B

$T_B + mg\cos 60° = m\dfrac{v_B^2}{r}$

$T_B = \dfrac{m}{a}v_B^2 - \frac{1}{2}mg$

According to Energy Equation

K.E at A + P.E at A = K.E at B + P.E at B

$\frac{1}{2}m(2\sqrt{ag})^2 + mg(a - a\cos 60°) = \frac{1}{2}mv_B^2 + mg(a + a\cos 60°)$

$2amg + \frac{1}{2}amg = \frac{1}{2}mv_B^2 + \frac{3}{2}amg$

$4ag + ag = v_B^2 + 3ag$

$v_B^2 = 2ag$

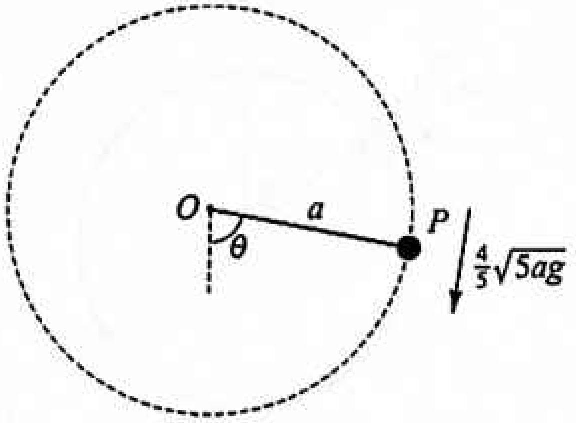
So $T_B = \dfrac{m}{a}\cdot 2ag - \frac{1}{2}mg = \frac{3}{2}mg$

$T_A : T_B = \frac{9}{2}mg : \frac{3}{2}mg$

$= 3 : 1$

Q.11 Oct/Nov/P31/2020

2



A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos\theta$. [5]

At Q $T = 0$

$mg = m\frac{v^2}{a}$

$v^2 = ag$

Energy Equation

K.E at Q + P.E at Q = K.E at P + P.E at P

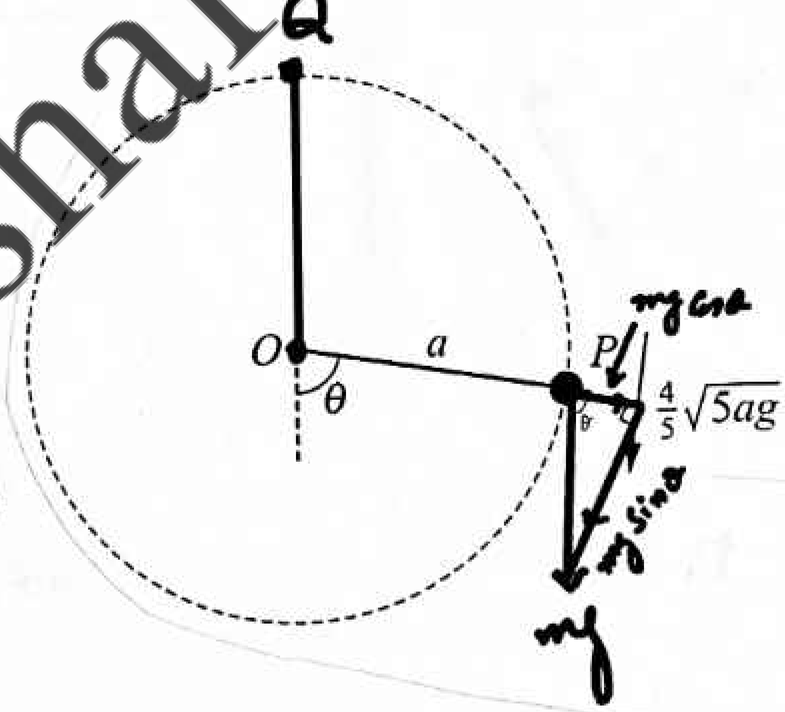
$\frac{1}{2}mv^2 + 2mga = \frac{1}{2}mu^2 + mg(a - a\cos\theta)$

$v^2 + 4ga = u^2 + 2ga - 2ga\cos\theta$

$ag + 4ag = \frac{16}{25} \cdot 5ag + 2ag - 2ag\cos\theta$

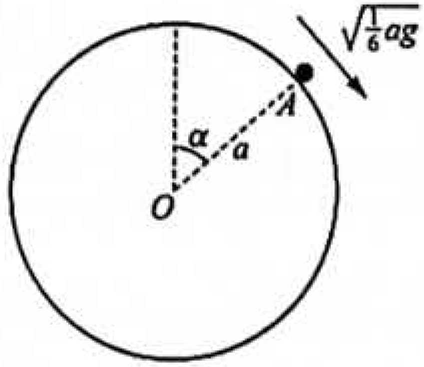
$2ag\cos\theta = \frac{1}{5}ag$

$\Rightarrow \cos\theta = \frac{1}{10}$



Q.12 Oct/Nov/P32/2020

1



A fixed smooth solid sphere has centre O and radius a. A particle of mass m is projected downwards with speed $\sqrt{\frac{1}{6}ag}$ from the point A on the surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B, where OB makes an angle β with the upward vertical through O.

Given that $\cos\alpha = \frac{2}{3}$, find the value of $\cos\beta$. [5]

At B

$mg\cos\beta = m\frac{v^2}{a}$

$v^2 = ag\cos\beta$ — (1)

K·E at A + P·E at A = K·E at B + K·E at B

$\frac{1}{2}mu^2 + mg(a + a\cos\alpha) = \frac{1}{2}mv^2 + mg(a + a\cos\beta)$

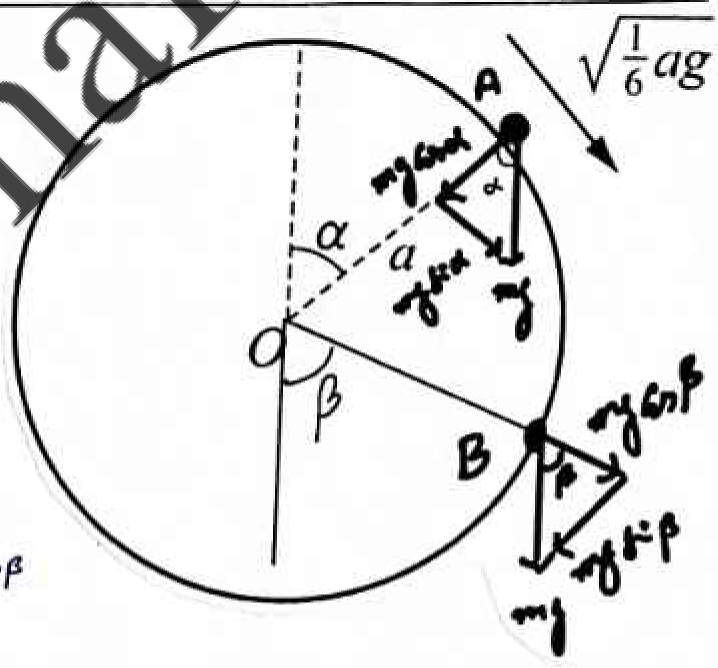
$u^2 + 2g(a + a\cos\alpha) = v^2 + 2g(a + a\cos\beta)$

$\frac{1}{6}ag + 2ag + 2ag\cos\alpha = ag\cos\beta + 2ag + 2ag\cos\beta$

$\frac{1}{6} + 2 + 2\cdot\frac{2}{3} = 3\cos\beta + 2$

$3\cos\beta = \frac{9}{6}$

$\cos\beta = \frac{1}{2}$



Q.13 Oct/Nov/P32+p33/2020

4 A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \frac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]

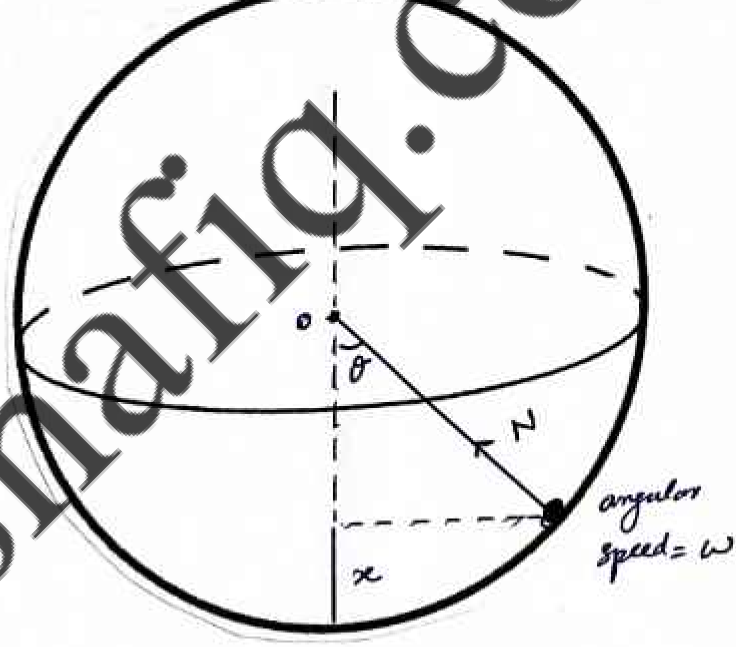
(a) Given angular speed $= \omega$

Radius of sphere $= r$

Let Normal reaction $= N$

then by resolving forces vertically

$N\cos\theta = mg$ — (1)



Resolving forces horizontally

$N\sin\theta = mr\sin\theta\,\omega^2$ — (2)

$N = mr\omega^2$

putting the value of N in (1)

$mr\omega^2\cos\theta = mg$

$\cos\theta = \frac{g}{\omega^2 r}$ — (3)

(b) when hight above the lowest point $= x$

then angular speed $= \omega$

so $\cos\theta = \frac{r-x}{r}$ — (4)

From (3) and (4)

$\frac{r-x}{r} = \frac{g}{\omega^2 r} \Rightarrow r - x = \frac{g}{\omega^2}$ — (5)

Given hight above the lowest point $= 4x$

then angular speed $= 2\omega$

$r - 4x = \frac{g}{4\omega^2}$

$4(r-4x) = \frac{g}{\omega^2}$ — (6)

From (5) and (6)

$r - x = 4(r - 4x)$

$x = \frac{1}{5}r$

Q.20 May/June/P21/2019

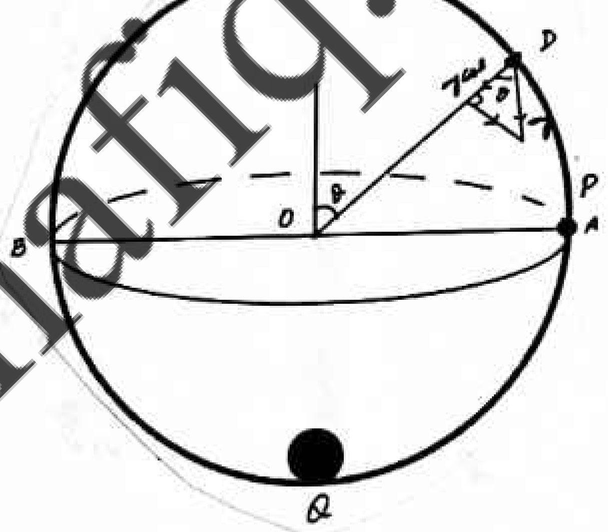
11

A particle P, of mass m, is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a. Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\left(\frac{21}{2}ag\right)}$ from A and begins to move in a vertical circle. At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q, of mass $4m$, and rebounds. The speed acquired by Q, as a result of the collision, is just sufficient for it to reach the point B.

(i) Find the speed of P and the speed of Q immediately after their collision. [7]

In its subsequent motion, P loses contact with the inner surface of the sphere at the point D, where the angle between OD and the upward vertical through O is θ.

(ii) Find $\cos\theta$. [5]



(i) Let u_P = speed of P at lowest point

v_Q = speed of Q immediately after the collision

By Conservation of Energy at lowest point

$$\frac{1}{2}mu_P^2 = \frac{1}{2}\left(\sqrt{\frac{21}{2}ag}\right)^2 + mga$$

$$u_P^2 = \frac{25}{2}ag$$

$$\frac{1}{2}4mv_Q^2 = 4mga$$

$$v_Q = \sqrt{2ag}$$

By using Conservation of momentum

$$mu_P = mv_P + 4mv_Q$$

(v_P = velocity of P immediately collision)

$$v_P = 4mv_Q - u_P$$

$$= 4\sqrt{2ag} - \frac{5}{\sqrt{2}}\sqrt{ag}$$

$$= \left[4\sqrt{2} - \frac{5\sqrt{2}}{2}\right]\sqrt{ag}$$

$$= \frac{3\sqrt{2}}{2}\sqrt{ag}$$

$$v_P = \frac{3}{\sqrt{2}}\sqrt{ag}$$

(ii) Let w_p = speed of P when it loses contact

By Energy equations

$$\frac{1}{2} m w_p^2 + mg(a + a\cos\theta) = \frac{1}{2} m v_p^2$$

$$w_p^2 = v_p^2 - 2\cancel{m}g(a + a\cos\theta)$$

$$w_p^2 = \frac{9}{2}ag - 2ga(1+\cos\theta)$$

$$= \left(\frac{5}{2} - 2\cos\theta\right)ag$$

Now apply $F = ma$

$$mg\cos\theta = m\frac{w_p^2}{a}$$

$$w_p^2 = ag\cos\theta$$

$$\left(\frac{5}{2} - 2\cos\theta\right)ag = ag\cos\theta$$

$$\frac{5}{2} - 2\cos\theta = \cos\theta$$

$$6\cos\theta = 5$$

$$\cos\theta = \frac{5}{6}$$

Q.21 May/June/P23/2019

2 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving in a complete vertical circle about O. The points A and B are on the circle, at opposite ends of a diameter, and such that OA makes an acute angle α with the upward vertical through O. The speed of P as it passes through A is $\frac{3}{2}\sqrt{(ag)}$. The tension in the string when P is at B is four times the tension in the string when P is at A.

(i) Show that $\cos\alpha = \frac{3}{4}$. [6]

(ii) Find the tension in the string when P is at B. [2]

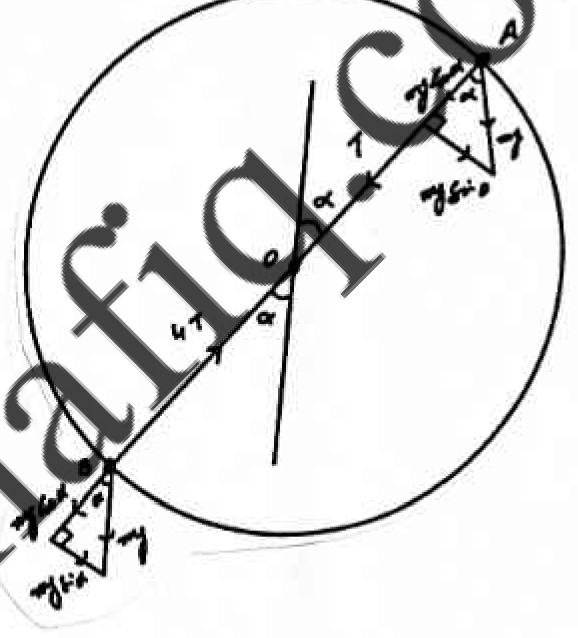
(a)

At A use F=ma

$T + mg\cos\alpha = m\left(\frac{3}{2}\sqrt{ag}\right)^2 / a$

$T + mg\cos\alpha = \frac{9}{4}mg$

$T = \frac{9}{4}mg - mg\cos\alpha$



At B again use F=ma

$4T - mg\cos\alpha = m\frac{v_B^2}{a}$

$4\left(\frac{9}{4}mg - mg\cos\alpha\right) - mg\cos\alpha = \frac{m}{a}v_B^2$

$9ag - 5ag\cos\alpha = v_B^2$

use Energy equation

KE at A + PE at A = KE at B + PE at B

$\frac{1}{2}m\left(\frac{3}{2}\sqrt{ag}\right)^2 + mg(a + a\cos\alpha) = \frac{1}{2}mv_B^2 + mg(a - a\cos\alpha)$

$\frac{9}{8}mag + mag + mag\cos\alpha = \frac{1}{2}m(9ag - 5ag\cos\alpha) + mag - mag\cos\alpha$

$\frac{9}{4} + 2 + 2\cos\alpha = 9 - 5\cos\alpha + 2 - 2\cos\alpha$

$9\cos\alpha = \frac{27}{4}$

$\cos\alpha = \frac{3}{4}$

(b) $T = \frac{9}{4}mg - \frac{3}{4}mg = \frac{6}{4}mg$

Tension at B $= 4\left(\frac{6}{4}mg\right) = 6mg$

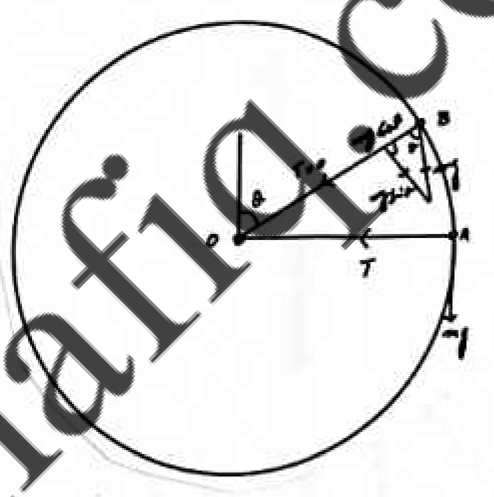
Q.22 Oct/Nov/P21+P22+P23/2019

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O.

(i) Show that $\cos\theta = \frac{2}{3}$. [5]

(ii) Find the greatest height, above the horizontal through O, reached by P in its subsequent motion. [4]

(i) Let B be the point when string becomes slack



At A use $F = ma$

$$T = \frac{m}{a}(\sqrt{2ag})^2$$

$$T = 2mg$$

At B

$$0 + mg\cos\theta = m\frac{v_B^2}{a}$$

$$v_B^2 = ag\cos\theta$$

According to Energy Equation

KE at A + PE at A = KE at B + PE at B

$$\frac{1}{2}m(2ag) + mga = \frac{1}{2}mv_B^2 + mg(a + a\cos\theta)$$

$$mag = \cancel{mag} = \frac{1}{2}m(ag\cos\theta) + \cancel{mag} + mag\cos\theta$$

$$1 = \frac{1}{2}\cos\theta + \cos\theta$$

$$\frac{3}{2}\cos\theta = 1$$

$$\cos\theta = \frac{2}{3}$$

(ii) Let V be the vertical velocity, then

$$V = V_B\sin\theta = \sqrt{ag\cos\theta}\cdot\sin\theta = \sqrt{\frac{2}{3}ag}\cdot\frac{\sqrt{5}}{3} = \sqrt{\frac{10}{27}ag}$$

use $2aS = V_f^2 - V_i^2$

$$-2gh = 0^2 - \left(\sqrt{\frac{10}{27}ag}\right)^2$$

→

$$h = \frac{\frac{10}{27} ag}{2g}$$

$$h = 0.185a$$

greatest hight from o $= h + a\cos\theta$

$= 0.185a + \frac{2}{3}a$

$= 0.852a$

Q.23 May/June/P21+P22/2018

11 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held so that the string is taut, with OP horizontal. The particle is projected downwards with speed $\sqrt{\left(\frac{2}{5}ag\right)}$ and begins to move in a vertical circle. The string breaks when its tension is equal to $\frac{11}{5}mg$.

(i) Show that the string breaks when OP makes an angle θ with the downward vertical through O, where $\cos\theta = \frac{3}{5}$. Find the speed of P at this instant. [6]

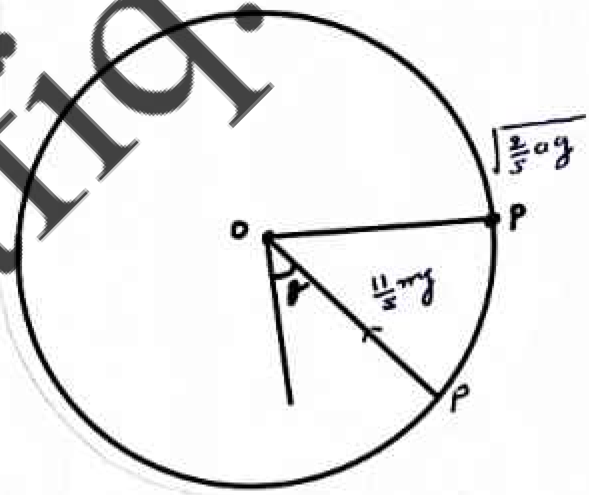
(ii) For the subsequent motion after the string breaks, find the distance OP when the particle P is vertically below O. [6]

(i) Let u = be the velocity at P

and V = be the velocity at P_1 (where string breaks)

By Energy equation

KE at P_1 + PE at P_1 = KE at P + PE at P



$\frac{1}{2}mv^2 + mg(a - a\cos\theta) = \frac{1}{2}mu^2 + mga$

$v^2 + 2ag - 2ag\cos\theta = u^2 + 2ag$

$v^2 = u^2 + 2ag\cos\theta \quad \text{---} (1)$

Given $T = \frac{11}{5}mg$ at P_1

use $F = ma$

$T - mg\cos\theta = \frac{mv^2}{a}$

$T = \frac{m}{a}(u^2 + 2ag\cos\theta) + mg\cos\theta$

$T = \frac{m}{a}u^2 + 2mg\cos\theta + mg\cos\theta$

$T = \frac{m}{a}u^2 + 3mg\cos\theta$

$\frac{11}{5}mg = \frac{m}{a}\cdot\frac{2}{5}ag + 3mg\cos\theta$

$\frac{11}{5} = \frac{2}{5} + 3\cos\theta$

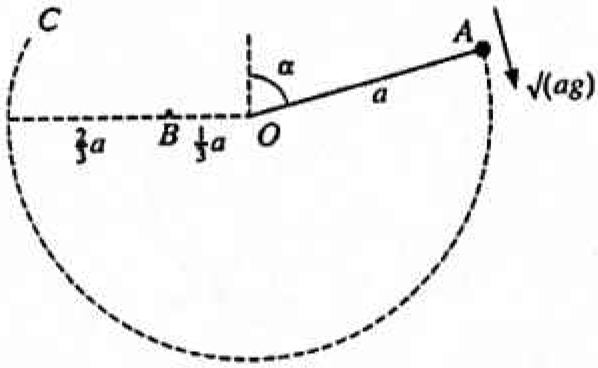
$3\cos\theta = \frac{9}{5}$

$\cos\theta = \frac{3}{5}$

So speed at P_1 $\quad v^2 = \frac{2}{5}ag + 2ag\left(\frac{3}{5}\right) = \frac{8}{5}ag \Rightarrow v = \sqrt{\frac{8}{5}ag}$

Q.24 May/June/P21+P22/2017

5



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that $OA = a$ and OA makes an angle α with the upward vertical through O. The particle is held at A and then projected downwards with speed $\sqrt{(ag)}$ so that it begins to move in a vertical circle with centre O. There is a small smooth peg at the point B which is at the same horizontal level as O and at a distance $\frac{1}{3}a$ from O on the opposite side of O to A (see diagram).

(i) Show that, when the string first makes contact with the peg, the speed of the particle is $\sqrt{(ag(1 + 2\cos\alpha))}$. [2]

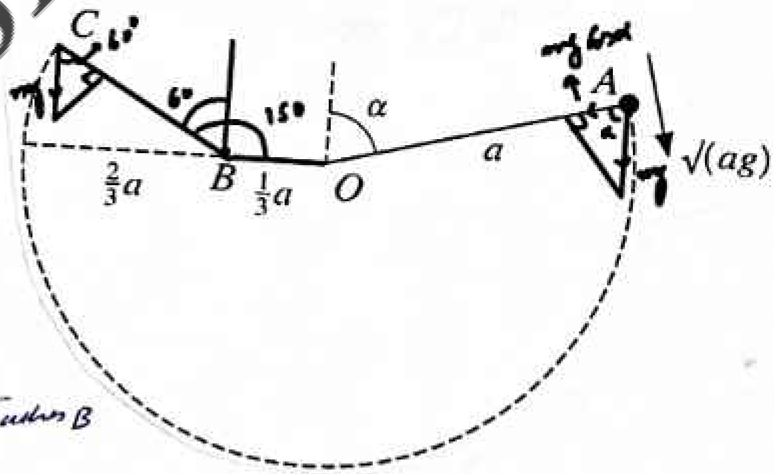
The particle now begins to move in a vertical circle with centre B. When the particle is at the point C where angle $CBO = 150°$, the tension in the string is the same as it was when the particle was at the point A.

(ii) Find the value of $\cos\alpha$. [10]

(i) Let u = be the velocity at A and given

$u = \sqrt{ag}$

and v_1 = be the velocity when string touches at B



According to Energy equation

KE at A + PE at A = KE at touches B + PE at touches B

$\frac{1}{2}m(\sqrt{ag})^2 + mg(a + a\cos\alpha) = \frac{1}{2}mv_1^2 + mga$

$\frac{1}{2}mag + mag + mag\cos\alpha = \frac{1}{2}mv_1^2 + mga$

$ag + 2ag\cos\alpha = v_1^2$

so $v_1 = \sqrt{ag(1 + 2\cos\alpha)}$

→

(ii) Let v_2 = be the velocity at C

T_A = be the tension at A

T_C = be the tension at C

use $F = ma$

$$T_A + mg\cos\alpha = m\frac{u^2}{a} = \frac{m\,ag}{a} = mg$$

$$T_A = mg(1-\cos\alpha)$$

According to Energy equations

KE at C + PE at C = KE at A + PE at A

$$\frac{1}{2}\cancel{m}v_2^2 + \cancel{m}g\left(a+\frac{2}{3}a\cos 60^\circ\right) = \frac{1}{2}\cancel{m}(\sqrt{ag})^2 + \cancel{m}g(a+a\cos\alpha)$$

$$v_2^2 + \cancel{2ag} + \frac{4}{3}ag\cdot\frac{1}{2} = ag + \cancel{2ag} + 2ag\cos\alpha$$

$$v_2^2 = ag\left(\frac{1}{3}+2\cos\alpha\right)$$

$$T_C + mg\cos 60^\circ = m\frac{v_2^2}{\frac{2}{3}a} = \frac{3m}{2a}v_2^2$$

$$= \frac{3m}{2a}\left(\frac{1}{3}+2\cos\alpha\right)ag$$

$$= \frac{1}{2}mg + 3mg\cos\alpha$$

$$T_C = 3mg\cos\alpha$$

Given $T_A = T_C$

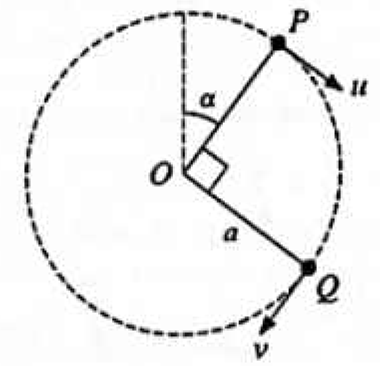
$$3mg\cos\alpha = mg - mg\cos\alpha$$

$$4\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{4}$$

Q.25 May/June/P23/2017

5



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is moving in complete vertical circles with the string taut. When the particle is at the point P, where OP makes an angle α with the upward vertical through O, its speed is u. When the particle is at the point Q, where angle $QOP = 90°$, its speed is v (see diagram). It is given that $\cos\alpha = \frac{4}{5}$.

(i) Show that $v^2 = u^2 + \frac{14}{5}ag$. [2]

The tension in the string when the particle is at Q is twice the tension in the string when the particle is at P.

(ii) Obtain another equation relating u^2, v^2, a and g, and hence find u in terms of a and g. [5]

(iii) Find the least tension in the string during the motion. [3]

(i) Given u = velocity at P

V = velocity at Q

According to Energy equation

KE at P + PE at P = KE at Q + PE at Q

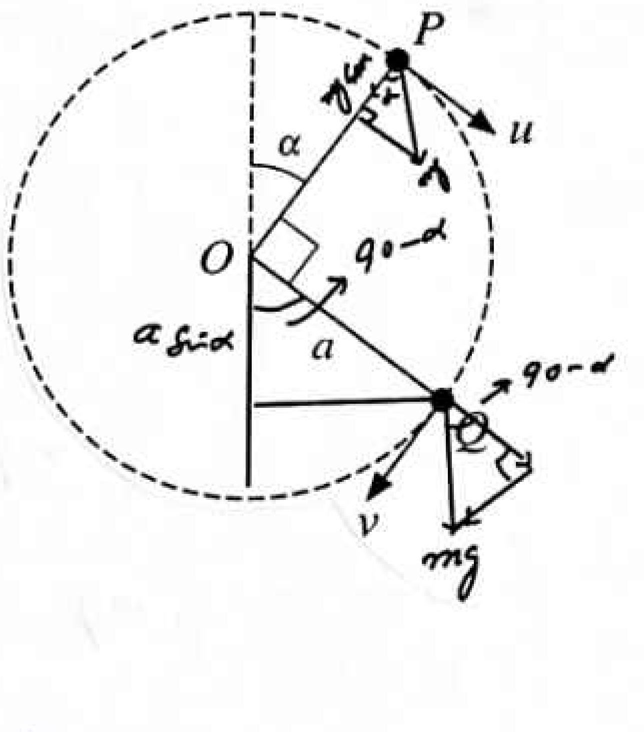
$\frac{1}{2}mu^2 + mg(a + a\cos\alpha) = \frac{1}{2}mv^2 + mg(a - a\sin\alpha)$

$u^2 + 2ag + 2ag\cos\alpha = v^2 + 2ag - 2ag\sin\alpha$

$v^2 = u^2 + 2ag(\cos\alpha + \sin\alpha)$

$v^2 = u^2 + 2ag\left(\frac{4}{5} + \frac{3}{5}\right)$ as 4 5 3

$v^2 = u^2 + \frac{14}{5}ag$ —①



(ii) use $F = ma$

$T_P + mg\cos\alpha = m\frac{u^2}{a} \Rightarrow T_P = \frac{m}{a}u^2 - \frac{4}{5}mg$

$T_Q - mg\sin\alpha = \frac{mv^2}{a} \Rightarrow T_Q = \frac{m}{a}v^2 + \frac{3}{5}mg$

Given $T_Q = 2T_P$ →

$$\frac{m}{a}v^2 + \frac{3}{5}mg = \frac{2m}{a}u^2 - \frac{8}{5}mg$$

$$v^2 + \frac{3}{5}ag = 2u^2 - \frac{8}{5}ag$$

$$v^2 = 2u^2 - \frac{11}{5}ag$$

$$u^2 + \frac{14}{5}ag = 2u^2 - \frac{11}{5}ag \quad \text{by equation ①}$$

$$u^2 = \frac{25ag}{5}$$

$$u = \sqrt{5ag}$$

(iii) Tension will be least at top $= T_{min}$, let ω = be the velocity at top

use $F = ma$

$$T_{min} + mg = m\frac{\omega^2}{a} \quad \text{— ②}$$

According to Energy equation

$$\frac{1}{2}m\omega^2 + mg(2a) = \frac{1}{2}mu^2 + mg(a + a\cos\alpha)$$

$$\omega^2 + 4ag = u^2 + 2ag + 2ag\cdot\frac{4}{5}$$

$$\omega^2 = 5ag - 2ag + \frac{8}{5}ag$$

$$\omega^2 = \frac{23}{5}ag \quad \text{put in ②}$$

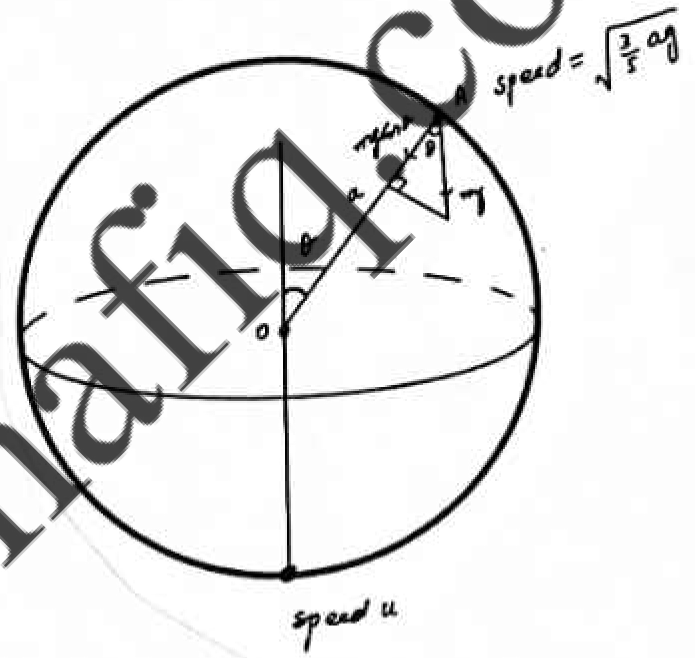
$$T_{min} + mg = \frac{23}{5}mg$$

$$T_{min} = \frac{18}{5}mg$$

Q.26 May/June/P21+P22+P23/2016

4 A particle P is at rest at the lowest point on the smooth inner surface of a hollow sphere with centre O and radius a. The particle is projected horizontally with speed u and begins to move in a vertical circle on the inner surface of the sphere. The particle loses contact with the sphere at the point A, where OA makes an angle θ with the upward vertical through O. Given that the speed of P at A is $\sqrt{(\frac{3}{5}ag)}$, find u in terms of a and g. [5]

Find, in terms of a, the greatest height above the level of O achieved by P in its subsequent motion. (You may assume that P achieves its greatest height before it makes any further contact with the sphere.) [5]



(a) Let V = be the speed at A, and given

$V = \sqrt{\frac{3}{5}ag}$

u = be the speed at lowest point

By Energy equation

$\frac{1}{2}mV^2 + mg(a + a\cos\theta) = \frac{1}{2}mu^2$

$\frac{1}{2}\cdot\frac{3}{5}ag + ag + ag\cos\theta = \frac{1}{2}u^2$

$u^2 = \frac{3}{5}ag + 2ag + 2ag\cos\theta$

use $F = ma$ at A

$mg\cos\theta = m\frac{V^2}{a}$

$mg\cos\theta = \frac{m}{a}\cdot\frac{3}{5}ag$

$\cos\theta = \frac{3}{5}$

put the value of $\cos\theta$ in above

$u^2 = \frac{3}{5}ag + 2ag + \frac{6}{5}ag$

$u^2 = \frac{19}{5}ag$

$u = \sqrt{\frac{19}{5}ag}$

(b) Let W = be the vertical velocity at A

$W = V\sin\theta = V\cdot(\frac{4}{5}) = \frac{4V}{5}$

Let h_A = greatest height above A

use $2aS = V^2 - u^2$

$-2gh_A = 0 - W^2$

$h_A = \frac{W^2}{2g} = \left(\frac{4V}{5}\right)^2 \div 2g$

$= \frac{16}{25}\cdot\frac{3}{5}ag \times \frac{1}{2g} = \frac{24}{125}a$

h_0 = greatest height above O

$h_0 = h_A + a\cos\theta = \frac{24}{125}a + \frac{3}{5}a = \frac{99}{125}a$