AS Level Further Mathematics

Topic: Hooks Law

Teacher:

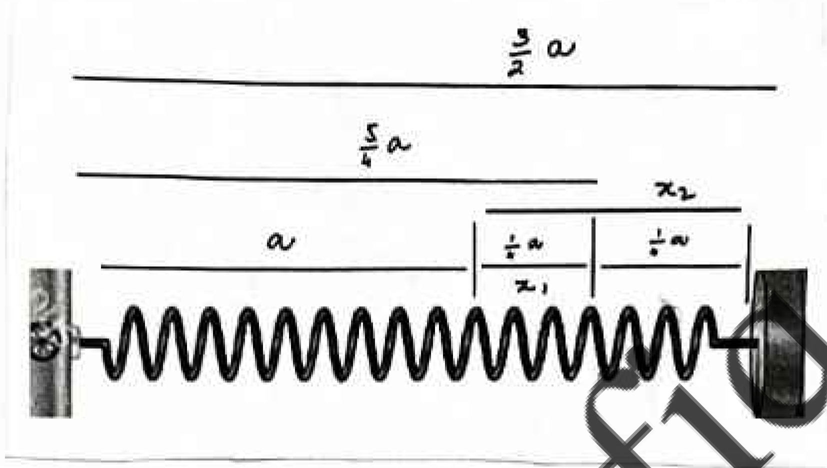
Muhammad Shafiq ur Rehman

Aitchison College Lahore

2 A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

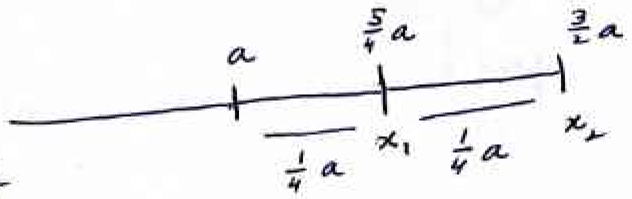
(a) Find an expression for v in terms of a and g. [4]

(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]



(a) Given $l = a$, $\lambda = 4mg$

Loss in K.E = Gain in EPE



$$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{\lambda x^2}{2l}$$

$$\frac{1}{2}mv^2 - \frac{1}{8}mv^2 = \frac{\lambda x_2^2}{2l} - \frac{\lambda x_1^2}{2l}$$

$$\frac{3}{8}mv^2 = \frac{\lambda}{2l}\left(x_2^2 - x_1^2\right)$$

$$\frac{3}{8}mv^2 = \frac{4mg}{2a}\left[\left(\frac{1}{2}a\right)^2 - \left(\frac{1}{4}a\right)^2\right]$$

$$\frac{3}{8}mv^2 = \frac{2mg}{a} \cdot \frac{3}{16}a^2$$

$$v^2 = ag$$

$$v = \sqrt{ag}$$

→

(b)

$$T = \frac{\lambda x}{a}$$

$$T = \frac{4mg}{a}\left(\frac{1}{2}a\right)$$

$$T = 2mg$$

use $F = ma$

$$T = ma$$

$$2mg = ma$$

$$a = 2g$$

3 One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O. A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v.

Find an expression for v in terms of g and a. [6]

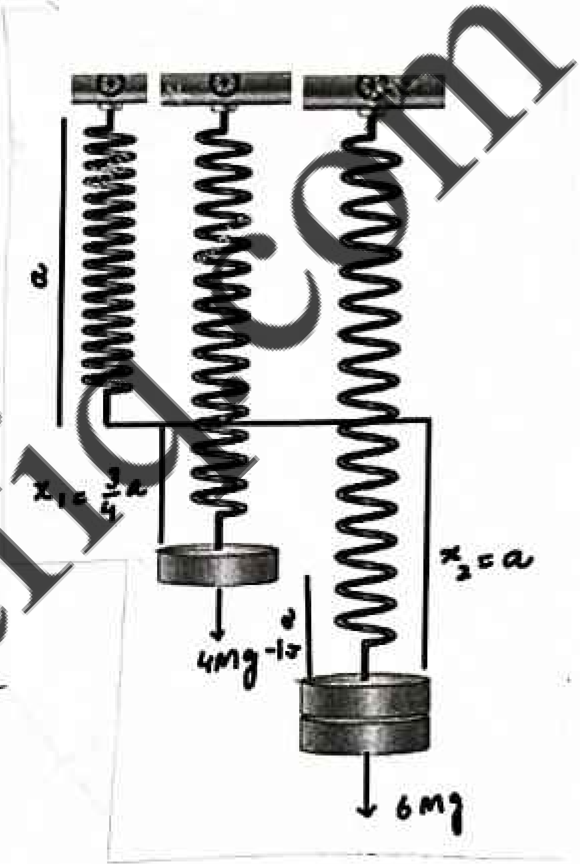
Given $l = a$, $\lambda = \frac{16}{3}Mg$

use

$$T = \frac{\lambda x}{l}$$

$$4Mg = \frac{\frac{16}{3}Mg\,x_1}{a}$$

$$x_1 = \frac{3}{4}a$$



when combined particle moves, then

Loss in GPE = gain in EPE + gain in K.E

$$(6Mg)\left(\frac{1}{4}a\right) = \left[\frac{\lambda x_2^2}{2a} - \frac{\lambda x_1^2}{2a}\right] + \frac{1}{2}6Mv^2$$

$$= \frac{\lambda}{2a}\left[x_2^2 - x_1^2\right] + 3Mv^2$$

$$= \frac{\frac{16}{3}Mg}{2a}\left[(a)^2 - \left(\frac{3}{4}a\right)^2\right] + 3Mv^2$$

$$= \frac{8Mg}{3a} \cdot \frac{7}{16}a^2 + 3Mv^2$$

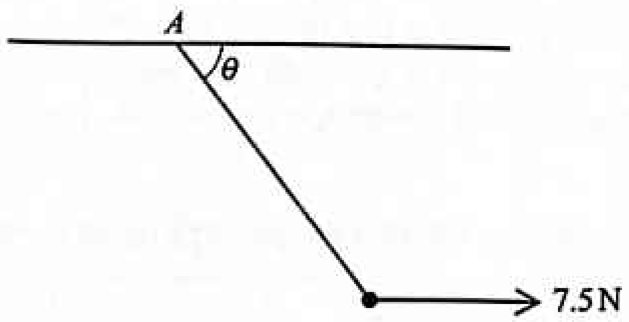
$$\frac{3Mga}{2} = \frac{7Mga}{6} + 3Mv^2$$

$$v^2 = \frac{1}{3}\left[\frac{3}{2}ga - \frac{7}{6}ga\right]$$

$$v^2 = \frac{1}{9}ga$$

$$v = \frac{1}{3}\sqrt{ga}$$

1



A particle of weight 10N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point A on a horizontal ceiling. A horizontal force of 7.5N acts on the particle. In the equilibrium position, the string makes an angle θ with the ceiling (see diagram). The string has natural length 0.8m and modulus of elasticity 50N.

(a) Find the tension in the string. **[2]**

(b) Find the vertical distance between the particle and the ceiling. **[3]**

(a)

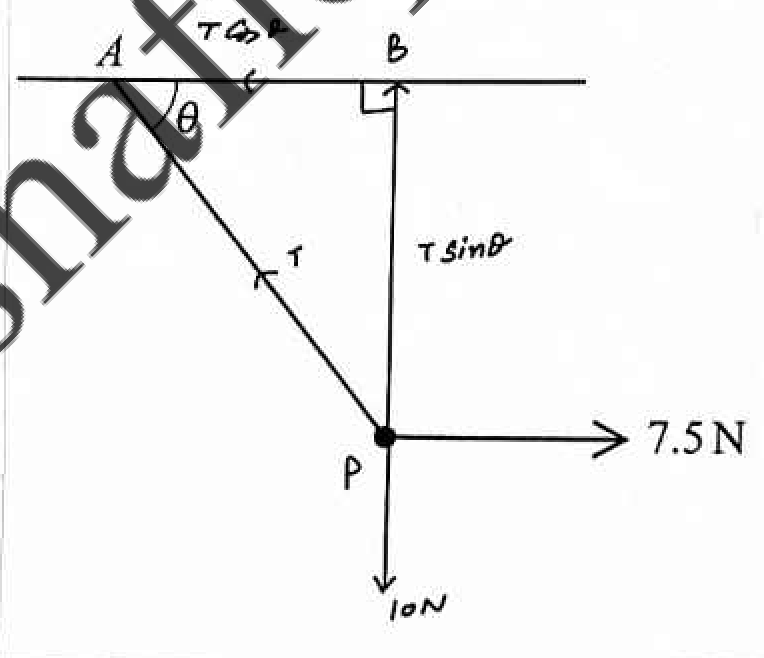
$\uparrow \quad T\sin\theta = 10 \quad \text{--- (1)}$

$\rightarrow \quad T\cos\theta = 7.5 \quad \text{--- (2)}$

$(1)^2 + (2)^2$

$T^2(\sin^2\theta + \cos^2\theta) = 10^2 + (7.5)^2$

$T = \sqrt{10^2 + (7.5)^2} = 12.5\text{N}$



(b) By Hooks law

$T = \frac{\lambda x}{a}$

$12.5 = \frac{50x}{0.8}$

$x = 0.2$

AP length $= 0.2 + 0.8 = 1$

From ① and ②

$\tan\theta = \frac{10}{7.5} \Rightarrow \theta = 53.1^\circ$

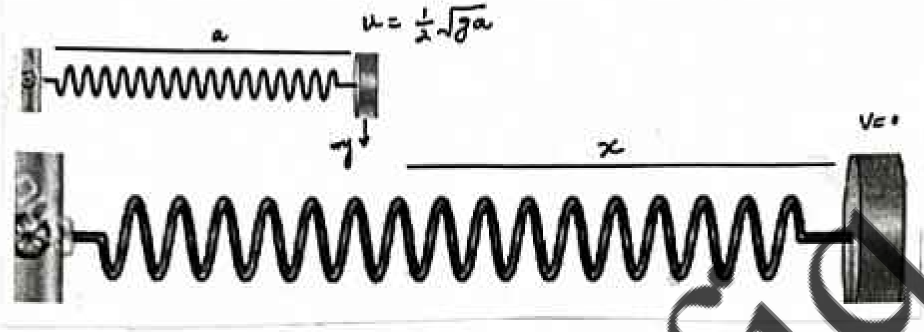
$\frac{PB}{AP} = \sin 53.1^\circ$

$PB = (AP)(\sin 53.1^\circ) = 1 \times \sin 53.1 = \frac{4}{5}$

Q.4 May/June/P33/2022

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]



Let x = be the greatest extension then

$$EPE = \frac{\lambda x^2}{2a} = \frac{\frac{4}{3}mgx^2}{2a} = \frac{2mgx^2}{3a}$$

By Energy equation

Loss in K.E = gain in EPE + work done against friction

$$\frac{1}{2}m\left(\frac{1}{2}\sqrt{ga}\right)^2 - 0 = \frac{2mgx^2}{3a} + \frac{1}{3}(mg)x$$

$$\frac{1}{8}a = \frac{2x^2}{3a} + \frac{x}{3}$$

$$3a^2 = 16x^2 + 8ax$$

$$16x^2 + 8ax - 3a^2 = 0$$

$$(4x - a)(4x + 3a) = 0$$

$$x = \frac{a}{4} \qquad x = -\frac{3a}{4} \text{ (Not possible)}$$

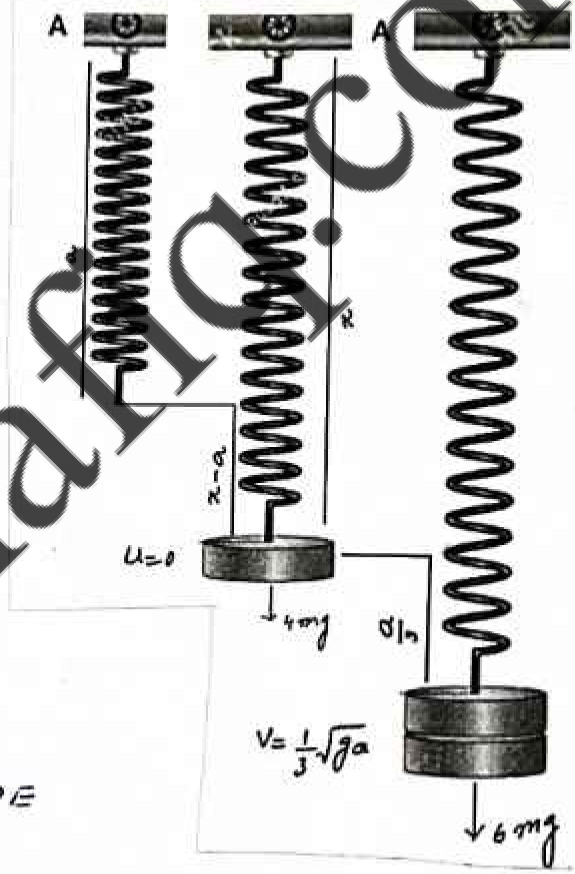
Q.5 May/June/P31+P32/2021

3 One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \dfrac{4a}{x-a}$. [1]

An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [6]



(a) Given $l = a$ $\quad \lambda = kmg$

By Hooks law

$$T = \frac{\lambda x}{a}$$

$$4mg = \frac{kmg(x-a)}{a}$$

$$4a = k(x-a)$$

$$k = \frac{4a}{x-a}$$

(b) Due to Energy equation

Gain in K.E + gain in EPE = Loss in PE

$$\frac{1}{2} \times 6m \times \frac{ga}{9} + \frac{kmg}{2a}\left[\left(x-a+\frac{a}{3}\right)^2 - (x-a)^2\right] = 6mg \times \frac{a}{3}$$

$$\frac{a}{3} + \frac{k}{2a}\left[x^2 - \frac{4}{3}xa + \frac{4a^2}{9} - x^2 + 2ax - a^2\right] = 2a$$

$$\frac{4a}{x-a} \cdot \frac{1}{2a}\left[\frac{2}{3}xa - \frac{5}{9}a^2\right] = 2a - \frac{a}{3}$$

$$2\left[\frac{(6x-5)a}{9}\right] = \frac{5}{3}a(x-a)$$

$$6x - 5a = \frac{5}{3} \times \frac{9}{2}$$

$$6x - 5a = \frac{15}{2}(x-a)$$

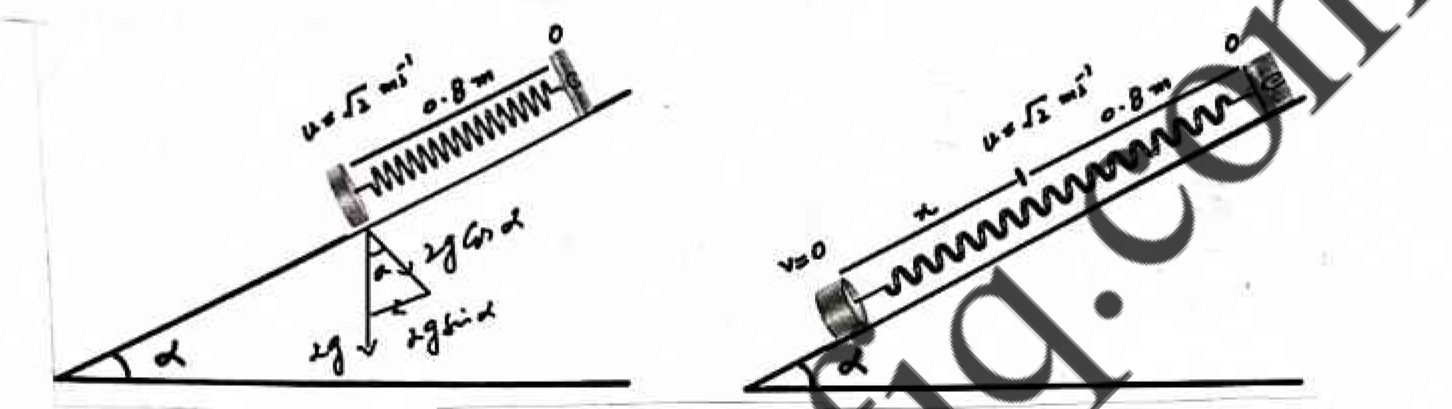
$$6x - \frac{15}{2}x = -\frac{15}{2}a$$

$$-3x = -5a \Rightarrow x = \frac{5}{3}a$$

Q.6 May/June/P33/2021

2 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\mathrm{m\,s^{-1}}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]



Given $l = 0.8\,m$ $\lambda = 36N$ $m = 2\,kg$ $u = \sqrt{2}\,ms^{-1}$, $\sin\alpha = \frac{3}{5}$

Due to Energy equation

Loss in GPE + Loss in K.E = gain in EPE

$$mg(x\sin\alpha) + \frac{1}{2}m(\sqrt{2})^2 = \frac{36x^2}{2\times0.8}$$

$$(2)(10)\left(x\times\frac{3}{5}\right) + \frac{1}{2}\times2\times2 = \frac{36x^2}{1.6}$$

$$12x + 2 = \frac{45}{2}x^2$$

$$45x^2 - 24x - 4 = 0$$

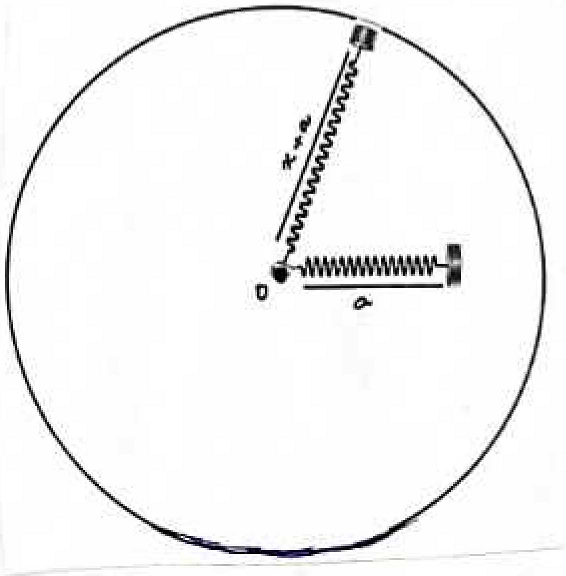
$$x = \frac{2}{3}, \quad x = \frac{-2}{15} \text{ (Not possible)}$$

$$x = \frac{2}{3} \text{ only}$$

Q.7 Oct/Nov/P31+P33/2021

1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]



Given $l = a \quad \lambda = 3mg$

Let x = be the extension

By Hooks law

$$T = \frac{\lambda x}{a}$$

$$T = \frac{3mgx}{a} \quad \text{—— ①}$$

Since particle moves in Horizontal circle

$$F = m\frac{v^2}{r}$$

$$T = m\left(\frac{4}{3}ga\right)\cdot\frac{1}{(x+a)}$$

$$T = \frac{4mga}{3(x+a)} \quad \text{—— ②}$$

From ① and ②

$$\frac{3mgx}{a} = \frac{4mga}{3(x+a)}$$

$$9x^2 + 9ax - 4a^2 = 0$$

$$(3x - a)(3x + 4a) = 0$$

$$x = \frac{a}{3}, \quad x = -\frac{4a}{3} \text{ (Not possible)}$$

$$x = \frac{a}{3} \text{ only}$$

Q.8 Oct/Nov/P31+P33/2021

3 A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

Given $l = a$ $\lambda = 12mg$

$$EPE = \frac{\lambda x^2}{2a}$$

$$\text{Loss in } EPE = \frac{1}{2} \times \frac{12mg e^2}{a} - \frac{1}{2} \times \frac{12mg}{a}\left(e - \frac{a}{3}\right)^2$$

$$\text{Loss in } EPE = \frac{6mge^2}{a} - \frac{6mg}{a}\left(e^2 - \frac{2}{3}ae + \frac{a^2}{9}\right)$$

$$= \frac{6mg}{a}\left[e^2 - e^2 + \frac{2}{3}ae - \frac{a^2}{9}\right]$$

$$= \frac{6mg}{a}\left[\frac{(6e-a)a}{9}\right]$$

$$= \frac{2mg}{3}(6e-a)$$

$$\text{Gain in } KE = \frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2ga})^2$$

$$= mga$$

$$\text{Gain in } GPE = mg\left(\frac{a}{3}\right) = \frac{mga}{3}$$

Due to energy equation

Gain in KE + Gain in PE = Loss in EPE

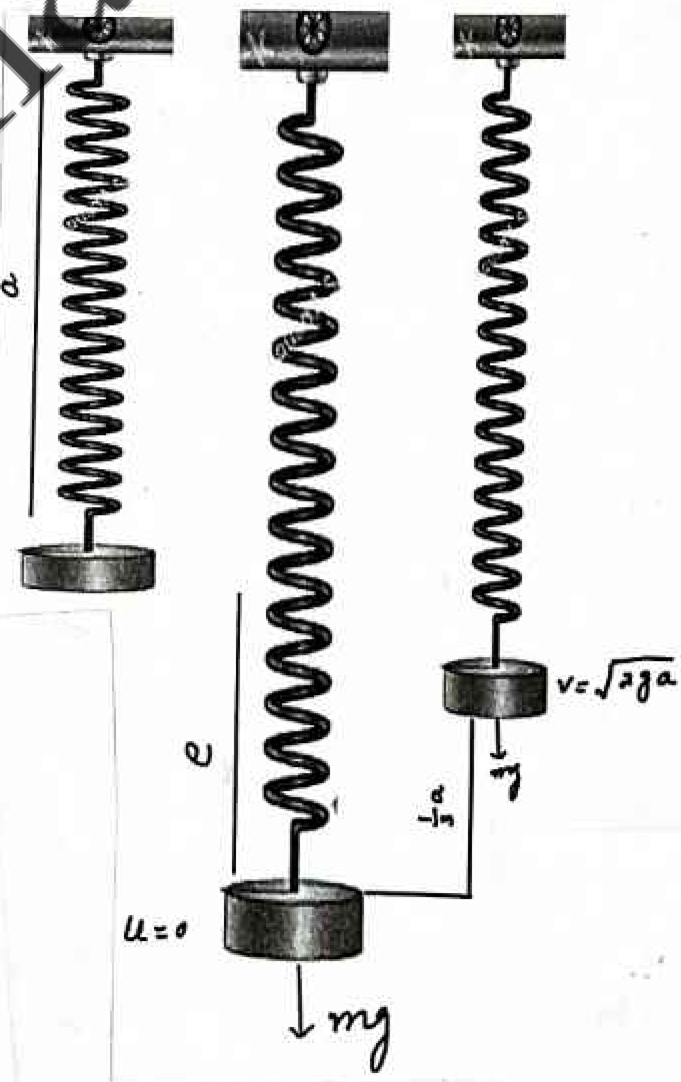
$$mga + \frac{mga}{3} = \frac{2}{3}mg(6e-a)$$

$$\frac{4}{3}a = \frac{2}{3}(6e-a)$$

$$6e - a = 2a$$

$$e = \frac{3a}{6}$$

$$e = \frac{1}{2}a$$



2 A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA. The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k. [6]

Given $l = a$ $\lambda = 5mg$ mass of $P = m$ mass of $Q = Km$

By Conservation law of Momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$\xleftarrow{+ve}$

$$m(0) + Km(\sqrt{4ga}) = (m + Km)V$$

$$V = \frac{Km\sqrt{4ga}}{m(1+K)} = \frac{K\sqrt{4ga}}{(1+K)}$$

Energy equation

Loss in KE = Gain in EPE

$$\frac{1}{2}(m + Km)V^2 = \frac{5mg}{2a}\left(\frac{a}{5}\right)^2 \quad \text{as } EPE = \frac{\lambda x^2}{2a}$$

$$\frac{1}{2}(1+K)m \cdot \frac{K^2(4ga)}{(1+K)^2} = \frac{mga}{10}$$

$$20K^2 = 1 + K$$

$$20K^2 - K - 1 = 0$$

$K = \frac{1}{4}$ $K = -\frac{1}{5}$ (Not possible)

3 One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

(b) Find the speed of P when the spring first returns to its natural length. [4]

(a) use $F = ma$

$T - mg = ma$

$T = mg + ma$ — (1)

by Hooks law

$T = \dfrac{\lambda x}{a}$

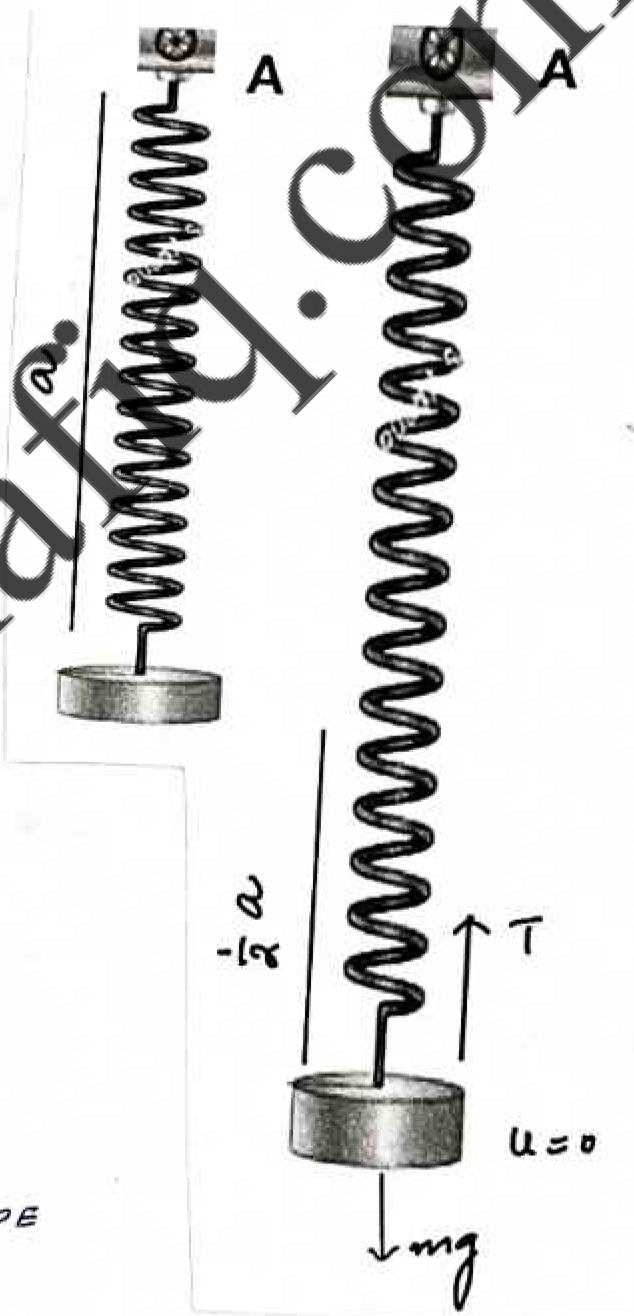
$T = \dfrac{5mg(\frac{1}{2}a)}{a} = \frac{5}{2}mg$ — (1)

From ① and ②

$mg + ma = \frac{5}{2}mg$

$a = \frac{5}{2}g - g$

$a = \frac{3}{2}g$



(b) Let $v =$ be the velocity when

$x =$

Energy equation

Gain in KE + Gain in GPE = Loss in EPE

$\frac{1}{2}mv^2 + mg(\frac{a}{2}) = \dfrac{5mg(\frac{a}{2})^2}{2a}$

$v^2 + ga = \frac{5}{4}ga$

$v^2 = \frac{1}{4}ga$

$v = \frac{1}{2}\sqrt{ga}$

Q.11 May/June/P33/2020

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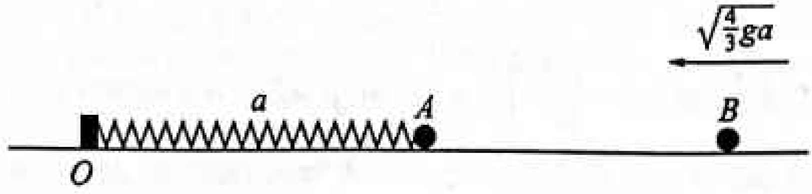


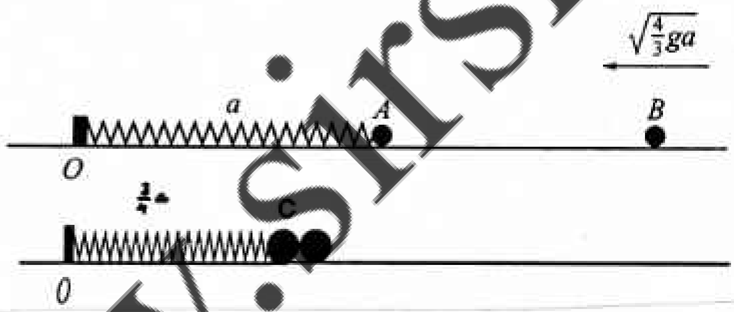
One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k. [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. [4]



(a) By conservation law of momentum

$+ve \leftarrow$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$km(0) + m\sqrt{\frac{4}{3}ga} = kmv + mv$$

$$v = \frac{\sqrt{\frac{4}{3}ga}}{(k+1)}$$

Energy equation

Loss in K.E = Gain in EPE

$$\frac{1}{2} m(k+1) V^2 - \frac{1}{2} m(k+1)\left(\frac{V}{2}\right)^2 = \frac{4mg\left(\frac{a}{4}\right)^2}{2a}$$

$$\frac{1}{2} m(k+1)\left[V^2 - \frac{V^2}{4}\right] = \frac{mg\,a^2}{8a}$$

$$\frac{1}{2} m(k+1) \cdot \frac{3}{4} V^2 = \frac{mg}{8}$$

$$\frac{1}{2}(k+1) \cdot \frac{3}{4} \cdot \frac{\frac{4}{3} ga}{(k+1)^2} = \frac{ga}{8}$$

$$\frac{1}{k+1} = \frac{1}{4}$$

$$k+1 = 4$$

$$k = 3$$

(b) Given $a = \frac{9}{20} g$

use $F = ma$

$$T + F = 4m\left(\frac{9}{20} g\right) \quad (F \text{ is force of friction})$$

$$\frac{\lambda x}{a} + \mu R = \frac{9}{5} mg \qquad \text{By Hooks law } T = \frac{\lambda x}{a}$$

$$\frac{4mg\left(\frac{a}{4}\right)}{a} + \mu(4mg) = \frac{9}{5} mg$$

$$1 + 4\mu = \frac{9}{5}$$

$$4\mu = \frac{4}{5}$$

$$\mu = \frac{1}{5}$$

1 A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

Let x = be the greatest extension

Gain in EPE $= \dfrac{\lambda x^2}{2a}$

$= \dfrac{3mg x^2}{2a}$

Loss in GPE $= mg(x \sin\theta)$

Energy Equation

Gain in EPE = Loss in GPE

$$\frac{3mg x^2}{2a} = mg x \sin\theta$$

$$x = \frac{2}{3} a \sin\theta$$

Q.13 Oct/Nov/P31/2020

3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

(b) Find the value of $\cos\theta$. [2]

Given $l = a$, $\lambda = 4mg$ mass $= m$

$x =$ extension in string

$= ka$

By Hooks law

$$T = \frac{\lambda x}{a} = \frac{4mg(ka)}{a}$$

$$T = 4kmg \quad \text{—} ①$$

Also given $\omega =$ angular speed

$$= \sqrt{\frac{g}{a}}$$

we know that

$$F = mr\omega^2$$

$$T\sin\theta = m(k+1)a\sin\theta\left(\sqrt{\frac{g}{a}}\right)^2$$

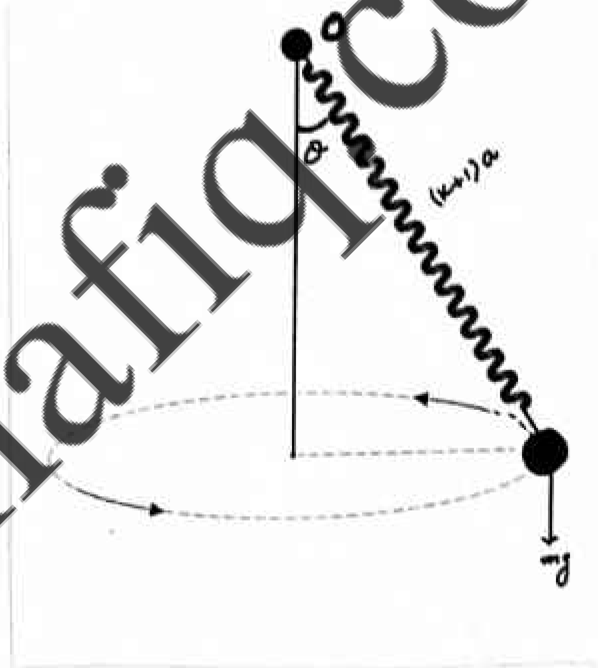
$$T = mg(k+1) \quad \text{—} ②$$

From ① and ②

$$4kmg = mg(k+1)$$

$$4k = k+1$$

$$k = \frac{1}{3}$$



(b) $T\cos\theta = mg$

$$\frac{4}{3}mg\cos\theta = mg$$

$$\cos\theta = \frac{3}{4}$$

Q.14 Oct/Nov/P32/2020

6 One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [8]

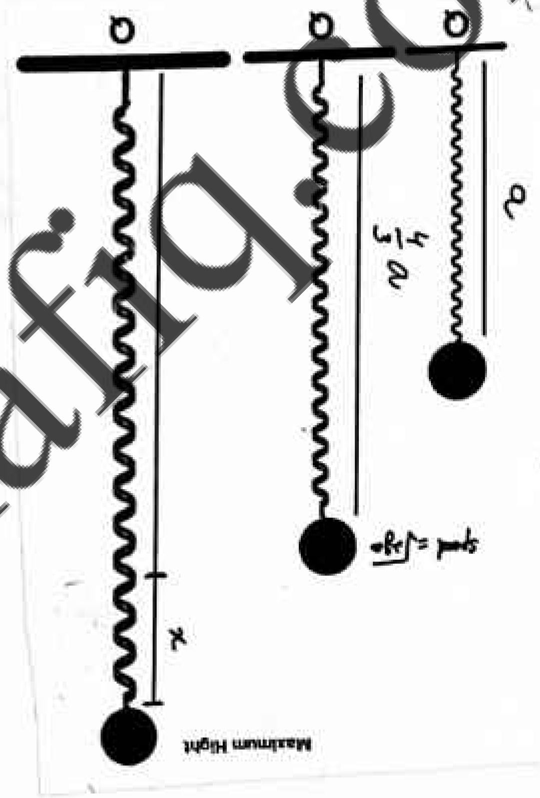
Given $l = a$ $\quad \lambda = k$ $\quad$ mass $= m$, $a = \frac{7}{3}g$ when speed $= \sqrt{2ga}$

At A

use $F = ma$

$$T + mg = m\left(\frac{7}{3}g\right)$$

$$T = \frac{4}{3}mg \quad \text{—} ①$$



By Hooks law

$$T = \frac{\lambda x}{a}$$

$$T = \frac{k\left(\frac{a}{3}\right)}{a} = \frac{k}{3} \quad \text{—} ②$$

From ① and ②

$$\frac{k}{3} = \frac{4}{3}mg$$

$$k = 4mg$$

Let $\frac{4}{3}a + x$ = be the greatest hight

At B

Gain in GPE $= mgx$

Loss in KE $= \frac{1}{2}m(2ga)$

Gain in EPE $= \frac{4mg}{2a}\left(x + \frac{a}{3}\right)^2 - \frac{4mg}{2a}\left(\frac{a}{3}\right)^2$

Energy equation Gain in EPE + Gain in GPE = loss in KE

$$\frac{4mg}{2a}\left[x^2 + \frac{2}{3}ax + \frac{a^2}{9} - \frac{a^2}{9}\right] + mgx = mga$$

$$2x^2 + \frac{7ax}{3} - a^2 = 0 \Rightarrow \boxed{x = \frac{1}{3}a} \quad \text{or } x = -\frac{3}{2}a \text{ (Not possible)}$$

Max hight $= \frac{4}{3}a + \frac{1}{3}a = \frac{5}{3}a$

Q.16 Oct/Nov/P21+P22+P23/2019

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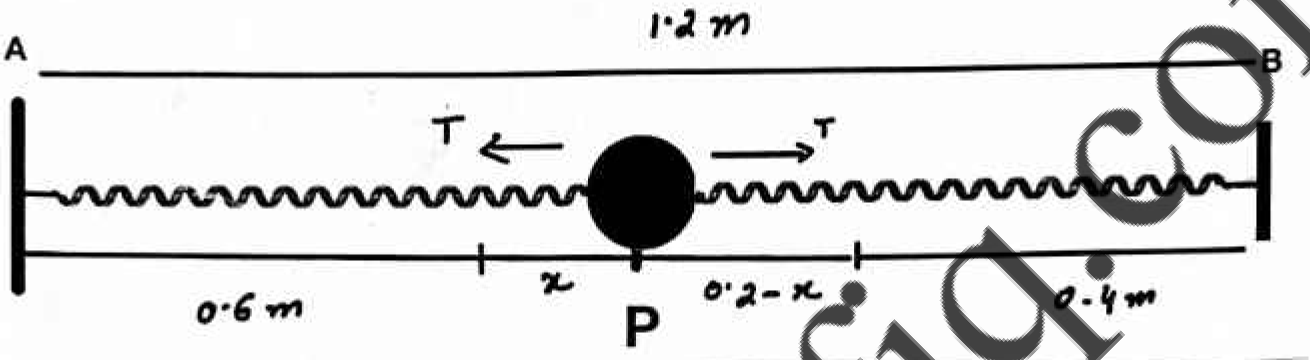
The points A and B are a distance 1.2 m apart on a smooth horizontal surface. A particle P of mass $\frac{2}{3}$ kg is attached to one end of a light spring of natural length 0.6 m and modulus of elasticity 10 N. The other end of the spring is attached to the point A. A second light spring, of natural length 0.4 m and modulus of elasticity 20 N, has one end attached to P and the other end attached to B.

(i) Show that when P is in equilibrium $AP = 0.75$ m. [3]



$$T = \frac{\lambda x}{a} \quad \text{By Hooks Law}$$

For AP

$$T = \frac{10x}{0.6} \quad \text{— ①}$$

For PB

$$T = \frac{20(0.2 - x)}{0.4} \quad \text{— ②}$$

From ① and ②

$$\frac{10x}{0.6} = \frac{20(0.2 - x)}{0.4}$$

$$4x = 12(0.2 - x)$$

$$4x = 2.4 - 12x$$

$$16x = 2.4$$

$$x = 0.15$$

$$AP = 0.6 + 0.15 = 0.75 \text{ m}$$

Q.15 May/June/P23/2019

A light spring has natural length a and modulus of elasticity kmg. The spring lies on a smooth horizontal surface with one end attached to a fixed point O. A particle P of mass m is attached to the other end of the spring. The system is in equilibrium with $OP = a$. The particle is projected towards O with speed u and comes to instantaneous rest when $OP = \frac{3}{4}a$.

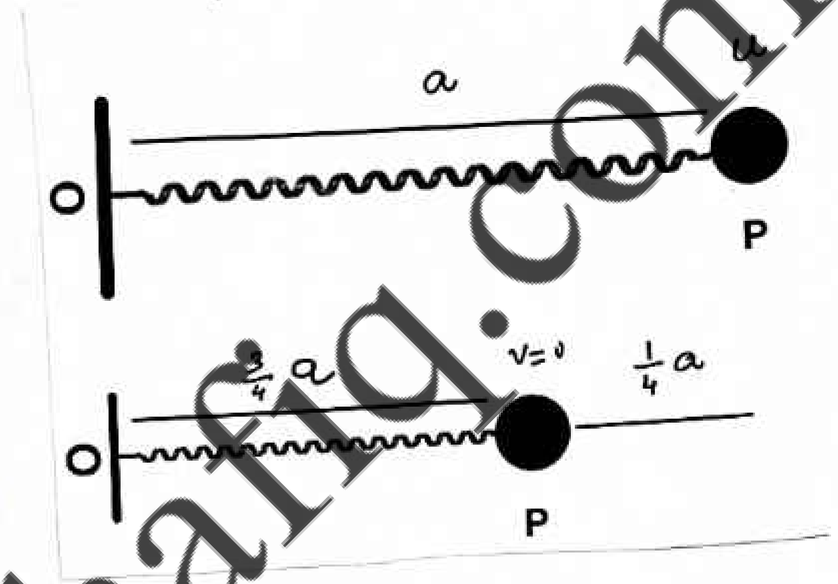
(i) Use an energy method to show that $k = \dfrac{16u^2}{ag}$. [2]

Given $l = a$ $\lambda = kmg$

Initial speed $= u$

compression $= \frac{1}{4}a$

then speed $= 0$



Energy equation

Gain in EPE = Loss of KE

$$\frac{kmg}{2a}\left(\frac{1}{4}a\right)^2 = \frac{1}{2}mu^2$$

$$\frac{kmga}{32} = \frac{1}{2}mu^2$$

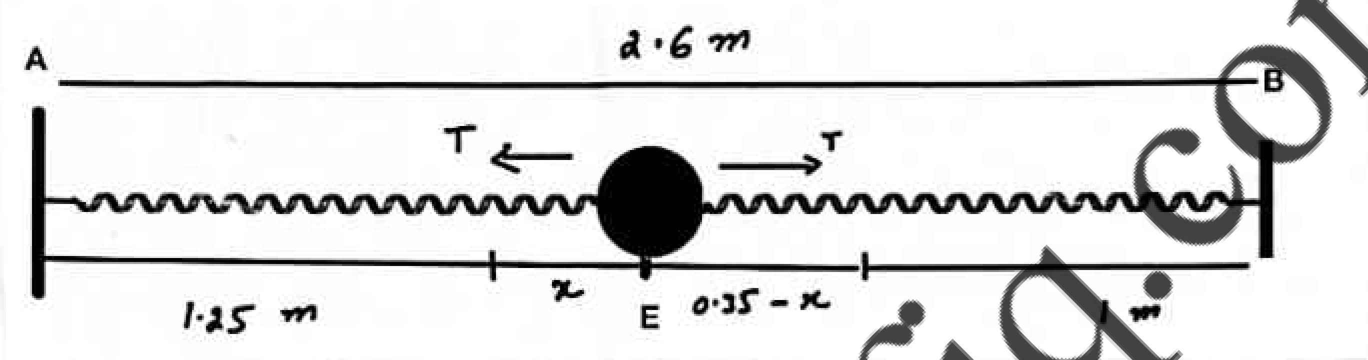
$$k = \frac{1}{2}mu^2 \times \frac{32}{mga}$$

$$k = \frac{16u^2}{ag}$$

Q.17 Oct/Nov/P21+P22+P23/2018

5 The fixed points A and B are on a smooth horizontal surface with $AB = 2.6$ m. One end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N, is attached to A. The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B; its other end is attached to P. The system is in equilibrium with P on the surface at the point E.

(i) Show that $AE = 1.4$ m. [4]



By Hooks law

$$T = \frac{\lambda x}{a}$$

For AE

$$T = \frac{\lambda x}{1.25} \quad — (1)$$

For EB

$$T = \frac{0.6\lambda(0.35-x)}{1} \quad — (2)$$

From (1) and (2)

$$\frac{\lambda x}{1.25} = \frac{0.6\lambda(0.35-x)}{1}$$

$$\frac{x}{1.25} = 0.6(0.35-x)$$

$$x = \frac{3}{4}(0.35-x)$$

$$4x = 1.05 - 3x$$

$$7x = 1.05$$

$$x = 0.15$$

$$AE = 1.25 + x = 1.25 + 0.15 = 1.4\,m$$