# 📘 IGCSE MATH NOTES – **Surds**

## 📌 1. Understand and Use Surds, Including Simplifying Expressions

### ✅ What is a Surd?

A **surd** is an **irrational number** that can't be simplified to remove the square root (or cube root, etc.). For example:

- √2, √3, √5 are **surds** because they give non-recurring, non-terminating decimals.
- √4 = 2 is **not** a surd because it simplifies to a rational number.

### ✅ Properties of Surds

These are essential for simplification:

- √a × √b = √(a × b)
- √a ÷ √b = √(a ÷ b)
- $(\sqrt{a})^2 = a$

### ✏️ Example 1: Simplify √50

**Solution:**

Break √50 into factors, one of which is a square number:

$$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

### ✏️ Example 2: Simplify 3√12 + 2√27

**Solution:**

First simplify each surd:

- √12 = √(4×3) = 2√3 → 3√12 = 3×2√3 = 6√3
- √27 = √(9×3) = 3√3 → 2√27 = 2×3√3 = 6√3

Now add:

$$6\sqrt{3} + 6\sqrt{3} = 12\sqrt{3}$$

✏️ **Example 3: Simplify (√3 + √2)(√3 - √2)**

**Solution:**

Use the identity:

$$(a+b)(a-b) = a^2 - b^2$$

So:

$$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$$

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## 📌 2. Rationalise the Denominator

When a surd is in the denominator, we "rationalise" it to remove the surd.

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### ✅ Case 1: Denominator is a single surd (e.g., $\frac{1}{\sqrt{2}}$)

Multiply **numerator and denominator** by the same surd.

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✏️ **Example 4: Rationalise $\frac{1}{\sqrt{2}}$**

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

### ✅ Case 2: Denominator is of the form $a + \sqrt{b}$ or $a - \sqrt{b}$

Multiply by the **conjugate**:

Conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$

### ✏️ Example 5: Rationalise $\frac{1}{2+\sqrt{3}}$

Multiply numerator and denominator by the conjugate: $2-\sqrt{3}$

$$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

Use identity:

$$(2+\sqrt{3})(2-\sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

So:

$$\frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

### ✏️ Example 6: Rationalise $\frac{3}{4-\sqrt{5}}$

Multiply by the conjugate $4+\sqrt{5}$:

$$\frac{3}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}} = \frac{3(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$

Denominator:

$$(4)^2 - (\sqrt{5})^2 = 16 - 5 = 11$$

Numerator:

$$3(4+\sqrt{5}) = 12 + 3\sqrt{5}$$

Answer:

$$\frac{12+3\sqrt{5}}{11}$$

## ✅ Summary Table

| Type | What to Do | Example Result |
|---|---|---|
| Simplify $\sqrt{ab}$ | Break into $\sqrt{a} \times \sqrt{b}$, simplify if possible | $\sqrt{50} = 5\sqrt{2}$ |
| Add/Subtract Surds | Simplify first, combine like surds | $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$ |
| Multiply Surds | Use distributive law or identities | $(\sqrt{3} + 1)(\sqrt{3} - 1) = 2$ |
| Rationalise ($\sqrt{a}$) denominator | Multiply by $\sqrt{a}/\sqrt{a}$ | $1/\sqrt{2} = \sqrt{2}/2$ |
| Rationalise ($a \pm \sqrt{b}$) | Multiply by conjugate | $1/(2 + \sqrt{3}) = 2 - \sqrt{3}$ |