

◆ 1. Simplify Expressions by Collecting Like Terms

Definition:

Combine terms that have the same variables raised to the same powers.

Example 1:

Simplify:

$$3x + 5x - 2x = (3 + 5 - 2)x = 6x$$

Example 2:

Simplify:

$$4a^2 + 3a - 2a^2 + 5a = (4a^2 - 2a^2) + (3a + 5a) = 2a^2 + 8a$$

◆ 2. Expand Products of Algebraic Expressions

a) Single Bracket Expansion

Use distributive property:

$$a(b + c) = ab + ac$$

Example:

$$3(x + 4) = 3x + 12$$

b) Double Bracket Expansion (FOIL method)

First, Outside, Inside, Last

Example:

$$(x + 3)(x + 2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

◆ 3. Factorise by Extracting Common Factors

Step:

Find the highest common factor (HCF) of all terms.

Example:

$$\text{Factorise: } 6x^2 + 9x = 3x(2x + 3)$$

◆ 4. Special Types of Factorisation

a) $ax + bx + kay + kby$

Group and factorise: Example:

$$ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$$

b) Difference of Squares:

$$a^2 - b^2 = (a - b)(a + b)$$

Example:

$$9x^2 - 25 = (3x - 5)(3x + 5)$$

c) Perfect Square Trinomials:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Example:

$$x^2 + 6x + 9 = (x + 3)^2$$

d) Quadratic Trinomials:

$$ax^2 + bx + c$$

Use factor pairs of $a \times c$ that add to b .

Example:

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Example:

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

e) Cubic Expressions (Group and factor):

Example:

$$x^3 + 3x^2 + 2x + 6$$

Group:

$$(x^3 + 3x^2) + (2x + 6) = x^2(x + 3) + 2(x + 3) = (x^2 + 2)(x + 3)$$

◆ 5. Complete the Square for Expressions in the form $ax^2 + bx + c$

If $a = 1$:

Example:

$$x^2 + 6x + 5 = (x + 3)^2 - 4$$

Steps:

1. Take half of **b**, square it: $(6/2)^2 = 9$
2. Add and subtract 9: $x^2 + 6x + 9 - 4 = (x + 3)^2 - 4$

If $a \neq 1$:

Example:

$$2x^2 + 8x + 7$$

Steps:

1. Factor out 2 from first two terms:
 $2(x^2 + 4x) + 7$
2. Complete the square inside:
 $2(x^2 + 4x + 4 - 4) + 7 = 2((x + 2)^2 - 4) + 7$
3. Simplify:
 $2(x + 2)^2 - 8 + 7 = 2(x + 2)^2 - 1$