Vectors in Two Dimensions: Notes and Examples

1. Translation Using a Vector

A translation in geometry refers to moving every point of an object (such as a shape) a certain distance in a specific direction. This movement can be described using a vector, where:

- A **vector** represents the displacement from one point to another.
- The vector can be written in component form as $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$, where x is the horizontal displacement, and y is the vertical displacement.

Example 1:

- If point $A(1,2)$ is translated by the vector $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, the new point A' after the translation will be at $(1+3, 2+4) = (4,6)$.
- Thus, the translation is represented as $A \to A'$ by the vector $\vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

2. Adding and Subtracting Vectors

- **Adding Vectors:** When two vectors are added, their components are added individually. If $\vec{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then the sum is:

$$\vec{u} + \vec{v} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

Example 2: If $\vec{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, then:

$$\vec{u} + \vec{v} = \begin{pmatrix} 2+1 \\ 3+(-4) \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Subtracting Vectors: To subtract vectors, subtract their corresponding components. If $\vec{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, then:

$$\vec{u} - \vec{v} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$$

Example 3: If $\vec{u} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, then:

$$\vec{u} - \vec{v} = \begin{pmatrix} 5-3 \\ 7-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

3. Multiplying a Vector by a Scalar

When a vector is multiplied by a scalar (a real number), each component of the vector is multiplied by the scalar. If $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ and the scalar is k, then:

$$k \cdot \vec{v} = \begin{pmatrix} k \cdot x \\ k \cdot y \end{pmatrix}$$

Example 4:

- If $\vec{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and the scalar is $k = 4$, then:

$$4 \cdot \vec{v} = 4 \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

This means the vector $\vec{v}$ has been scaled by a factor of 4.

Summary of Operations:

- **Translation**: Move a point using a vector.
- **Addition**: Add the corresponding components of two vectors.
- **Subtraction**: Subtract the corresponding components of two vectors.
- **Scalar Multiplication**: Multiply each component of a vector by a scalar.