📊 Algebraic Fractions – Detailed Notes and Examples

✅ 1. Manipulate Algebraic Fractions

Algebraic fractions are fractions where the numerator, the denominator, or both contain algebraic expressions (expressions with variables).

🔷 A. Adding and Subtracting Algebraic Fractions

Step 1: Find the **Lowest Common Denominator (LCD)**

Step 2: Make denominators the same

Step 3: Add or subtract the numerators

Step 4: Simplify if possible

Example 1:

$$\frac{2}{x} + \frac{3}{x} = \frac{2+3}{x} = \frac{5}{x}$$

Example 2:

$$\frac{1}{x} + \frac{2}{x+1} \Rightarrow \text{LCD} = x(x+1)$$

$$= \frac{(x+1)}{x(x+1)} + \frac{2x}{x(x+1)} = \frac{x+1+2x}{x(x+1)} = \frac{3x+1}{x(x+1)}$$

🔷 B. Multiplying and Dividing Algebraic Fractions

Multiplying:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Dividing:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Always factor and simplify before multiplying.

Example 3 (Multiplication):

$$\frac{x^2-4}{x^2-x-6} \cdot \frac{x-3}{x+2} \Rightarrow \text{Factor first:}$$

$$= \frac{(x-2)(x+2)}{(x-3)(x+2)} \cdot \frac{x-3}{x+2} \Rightarrow \text{Cancel common terms:}$$

$$= \frac{x-2}{x+2}$$

Example 4 (Division):

$$\frac{x^2-9}{x^2-1} \div \frac{x-3}{x-1} \Rightarrow \text{Rewrite as multiplication:}$$

$$= \frac{(x-3)(x+3)}{(x-1)(x+1)} \cdot \frac{x-1}{x-3} \Rightarrow \text{Cancel: } (x-3), (x-1)$$

$$= \frac{x+3}{x+1}$$

✅ 2. Factorise and Simplify Rational Expressions

To simplify rational expressions (fractions with algebraic expressions), factor the numerator and denominator fully and cancel any **common factors**.

◆ A. Factorising Techniques Review

1. **Common factors**
   $$2x^2 + 4x = 2x(x+2)$$

2. **Difference of squares**
   $$x^2 - 9 = (x-3)(x+3)$$

3. **Quadratic expressions**
   $$x^2 + 5x + 6 = (x+2)(x+3)$$

◆ **B. Simplify by Cancelling**

Example 5:

$$\frac{x^2 - 9}{x^2 - x - 6} = \frac{(x-3)(x+3)}{(x-3)(x+2)} = \frac{x+3}{x+2}$$

Example 6:

$$\frac{2x^2 + 8x}{x^2 + 2x} = \frac{2x(x+4)}{x(x+2)} = \frac{2(x+4)}{x+2}$$

◆ **C. Complex Fractions (Fractions within Fractions)**

Example 7:

$$\frac{\frac{1}{x} + \frac{1}{y}}{x+y} \Rightarrow \text{Find numerator first: } \frac{y+x}{xy} \Rightarrow \text{Now: } \frac{(x+y)/xy}{x+y} = \frac{1}{xy}$$

🔍 **Tips:**

- **Always factor before simplifying.**

- **Look for special patterns** like the difference of squares.

- **Cancel common factors** only if they multiply the whole numerator/denominator.