

Magnitude of a Vector – Notes and Examples

The **magnitude** (or **length**) of a vector represents the distance from the origin to the point represented by the vector. It's calculated using the **Pythagorean theorem** in 2D or 3D space.

Formula for the Magnitude of a Vector:

For a vector \mathbf{v} with components $\mathbf{v} = \langle x, y \rangle$ in two dimensions (2D), the magnitude is given by:

$$|\mathbf{v}| = \sqrt{x^2 + y^2}$$

In three dimensions (3D), for a vector $\mathbf{v} = \langle x, y, z \rangle$, the magnitude is:

$$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

Example 1 (2D Vector):

Given the vector $\mathbf{v} = \langle 3, 4 \rangle$, the magnitude is calculated as:

$$|\mathbf{v}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

So, the magnitude of the vector is 5.

Example 2 (3D Vector):

Given the vector $\mathbf{v} = \langle 1, -2, 2 \rangle$, the magnitude is calculated as:

$$|\mathbf{v}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

So, the magnitude of the vector is 3.

Magnitude Notation:

The magnitude of a vector \mathbf{a} is written as:

$$|\mathbf{a}|$$

For example, the magnitude of vector \mathbf{AB} is written as:

$$|\overrightarrow{AB}|$$

Key Notes:

1. The magnitude is always a non-negative scalar (positive or zero).
2. The magnitude is used to determine the length of a vector or the distance between two points in geometry or physics.

Applications:

- In physics, the magnitude of a vector often represents the **speed** or **distance**.
- In geometry, the magnitude of the displacement vector tells you how far one point is from another.