◆ 1. Sample Space Diagrams

A **sample space** shows all possible outcomes of an event.

Example 1: Tossing two coins

Sample space:

S = {HH, HT, TH, TT}

Let A = "at least one head"

A = {HH, HT, TH}

P(A) = 3/4

Let B = "both tails"

B = {TT}

P(B) = 1/4

Combined Events:

- **P(A ∩ B)** = probability of both A and B
  → A ∩ B = ∅ → P(A ∩ B) = 0
- **P(A ∪ B)** = probability of A or B or both
  → A ∪ B = {HH, HT, TH, TT} → P(A ∪ B) = 4/4 = 1

◆ 2. Venn Diagrams

Used to represent **events and their relationships.**

Example 2:

In a class of 40 students:

- 25 study Math (M),
- 18 study Physics (P),
- 10 study both.

Let's use a Venn diagram to find:

- **P(M ∪ P)** (Math or Physics)
- **P(M ∩ P)** (both Math and Physics)

We fill the Venn diagram:

- $M \cap P = 10$
- M only = 25 - 10 = 15
- P only = 18 - 10 = 8
- Outside both = 40 - (15 + 10 + 8) = 7

So:

- **$P(M \cap P)$** = 10/40 = 0.25
- **$P(M \cup P)$** = (15 + 10 + 8)/40 = 33/40 = 0.825

🔷 3. Tree Diagrams

Tree diagrams are great for **multi-stage experiments**, especially:

- With replacement → probabilities stay the same
- Without replacement → probabilities change

Example 3: With replacement

Bag has 3 red and 2 blue balls. Draw 2 balls **with replacement**.

Tree diagram:

- First draw: P(R) = 3/5, P(B) = 2/5
- Second draw (same probabilities since replacement)

To find:

P(both red) = P(R and R) = (3/5) × (3/5) = 9/25

P(one red, one blue) = P(R then B) + P(B then R)
= (3/5 × 2/5) + (2/5 × 3/5) = 6/25 + 6/25 = 12/25

Example 4: Without replacement

Same bag, but draw 2 balls **without replacement**

- First draw: P(R) = 3/5, P(B) = 2/5
- Second draw changes depending on the first

P(both red) = (3/5) × (2/4) = 6/20 = 3/10

P(one red, one blue) =

P(R then B) = (3/5) × (2/4) = 6/20

P(B then R) = (2/5) × (3/4) = 6/20

→ Total = 12/20 = 3/5

🔶 Notation Recap:

- **P(A ∩ B)** = Probability of A **and** B happening
- **P(A ∪ B)** = Probability of A **or** B or both happening