IGCSE Math – Equations Notes & Examples

1. Construct Expressions, Equations and Formulas

Expression: A combination of numbers, variables, and operators (e.g. $3x + 2$)

Equation: An expression that equals something (e.g. $3x + 2 = 8$)

Formula: A rule showing the relationship between variables (e.g. $A = \pi r^2$)

Example:

> The cost C of buying x pens at \$2 each:
> $C = 2x$

Constructing Example:

> Write an expression for the product of two consecutive even numbers:
> Let first even number = $2n$, next = $2n + 2$
> Product = $2n(2n + 2)$

2. Solve Linear Equations in One Unknown

Steps:

- Simplify both sides
- Move all terms with variables to one side
- Solve for the unknown

Example 1:

$3x + 4 = 10$

$\rightarrow 3x = 6$

$\rightarrow x = 2$

Example 2:

$5 - 2x = 3(x + 7)$

$\rightarrow 5 - 2x = 3x + 21$

$\rightarrow -2x - 3x = 21 - 5$

$\rightarrow -5x = 16$

$\rightarrow x = -\frac{16}{5}$

3. Solve Fractional Equations with Numerical and Linear Algebraic Denominators

Tip: Multiply through by the LCM to eliminate denominators.

Example 1:

$\frac{x}{2x+1} = 4$

→ Multiply both sides by $2x + 1$:

→ $x = 4(2x + 1)$

→ $x = 8x + 4$

→ $-7x = 4$

→ $x = -\frac{4}{7}$

Example 2:

$\frac{2}{x+2} + \frac{3}{2x-1} = 1$

→ LCM = $(x + 2)(2x - 1)$

Multiply everything by LCM and solve normally.

Example 3:

$\frac{x}{x+2} = \frac{3}{x-6}$

→ Cross multiply:

$x(x - 6) = 3(x + 2)$

→ Expand and solve.

4. Solve Simultaneous Linear Equations in Two Unknowns

Methods: Substitution or Elimination

Example (Elimination):

$2x + 3y = 12$

$4x - 3y = 6$

Add both:

$6x = 18$

$x = 3$, then substitute to find y

5. Solve Simultaneous Equations: One Linear, One Non-linear

Example:

$y = 2x + 1$ (linear)

$x^2 + y^2 = 13$ (non-linear)

→ Substitute y into the second:

$x^2 + (2x + 1)^2 = 13$

→ Expand and solve the quadratic.

6. Solve Quadratic Equations

a) By Factorisation

Example:

$x^2 + 5x + 6 = 0$

$\rightarrow (x + 2)(x + 3) = 0$

$\rightarrow x = -2$ or -3

b) By Completing the Square

Example:

$x^2 + 6x + 2 = 0$

$\rightarrow (x + 3)^2 - 9 + 2 = 0$

$\rightarrow (x + 3)^2 = 7$

$\rightarrow x = -3 \pm \sqrt{7}$

c) By Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$x^2 + 4x - 5 = 0$

$\rightarrow a = 1, b = 4, c = -5$

$\rightarrow x = \frac{-4 \pm \sqrt{16+20}}{2} = \frac{-4 \pm \sqrt{36}}{2} = -1, -5$

7. Change the Subject of a Formula

Example 1:

Make x the subject:

$y = 3x + 2$

$\rightarrow y - 2 = 3x$

$\rightarrow x = \frac{y-2}{3}$

Example 2 (subject appears twice):

$A = x + \frac{3}{x}$

$\rightarrow$ Multiply by x: $Ax = x^2 + 3$

$\rightarrow x^2 - Ax + 3 = 0$

$\rightarrow$ Solve using quadratic formula.

Example 3 (with square root):

$A = \sqrt{x + 3}$

$\rightarrow A^2 = x + 3$

$\rightarrow x = A^2 - 3$