🔺 1. Sine Rule

Used when:

- You have **two angles and one side** (AAS or ASA).
- You have **two sides and a non-included angle** (SSA, *ambiguous case*).

Formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Where:

- a, b, c are sides.
- A, B, C are opposite angles.

🔁 Ambiguous Case (SSA)

- If $\frac{a}{\sin A} = \frac{b}{\sin B}$ and A is **acute**, there can be:
  - **No triangle** (if side opposite the given angle is too short).
  - **One triangle** (if it just touches the base).
  - **Two triangles** (if it can swing to form two different angles).

📏 2. Cosine Rule

Used when:

- You know **two sides and the included angle** (SAS).
- You know **all three sides** (SSS), to find an angle.

Formula for side:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Formula for angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

📐 3. Area of a Triangle

Used when:

- You know **two sides and the included angle** (SAS).

Formula:

$$\text{Area} = \frac{1}{2}ab\sin C$$

Works for **any triangle**, even with **obtuse** angles (since $\sin$ still works with obtuse angles in radians/degrees).

✅ Examples

Example 1: Sine Rule (AAS)

Given: $A = 40^\circ, B = 60^\circ, a = 10$

Find: b

Use sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{10}{\sin 40^\circ} = \frac{b}{\sin 60^\circ} \Rightarrow b = \frac{10 \cdot \sin 60^\circ}{\sin 40^\circ}$$

Example 2: Cosine Rule (SSS)

Given: $a = 7, b = 8, c = 9$

Find: A

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{64 + 81 - 49}{2 \cdot 8 \cdot 9} = \frac{96}{144} = 0.6667 \Rightarrow A \approx \cos^{-1}(0.6667)$$