Inequalities - IGCSE Mathematics Notes

1. Represent and Interpret Inequalities on a Number Line

Inequality Symbols Recap:

Symbol	Meaning	Example	Words
$<$	Less than	$x < 3$	x is less than 3
$>$	Greater than	$x > 3$	x is greater than 3
$\leqslant$	Less than or equal	$x \leqslant 3$	x is less than or equal to 3
$\geqslant$	Greater than or equal	$x \geqslant 3$	x is greater than or equal to 3

Graphical Representation on a Number Line:

- Use **open circles (○)** for **strict inequalities:** $<$, $>$
- Use **closed circles (●)** for **inclusive inequalities:** $\leqslant$, $\geqslant$

Example 1: Represent the inequality:

$-3 \leqslant x < 1$

Number Line Representation:

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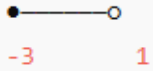

- Closed circle at –3 because of $\leqslant$
- Open circle at 1 because of $<$

2. Construct, Solve, and Interpret Linear Inequalities

To solve inequalities, treat them almost like equations, but **remember to reverse the inequality sign if you multiply or divide by a negative number.**

Steps:

1. Simplify both sides (like equations).
2. Solve for x.
3. Represent the solution if asked (e.g., on a number line).

Example 2:

Solve:

$3x < 2x + 4$

Solution:

- Subtract 2x from both sides:

  $x < 4$

This means any value of x less than 4 is a solution.

Example 3: Solve:

$-3 \leqslant 3x - 2 < 7$

Solution (step-by-step):

1. Solve the inequality as a **compound inequality**:

   $-3 \leqslant 3x - 2 < 7$

2. Add 2 to all parts:

   $-1 \leqslant 3x < 9$

3. Divide all parts by 3:

   $-1/3 \leqslant x < 3$

3. Represent and Interpret Linear Inequalities in Two Variables Graphically

A **linear inequality in two variables** looks like an equation but with an inequality, such as:

- $y > 2x + 1$
- $x \leqslant 4$
- $y < -x + 3$

Graphing Steps:

1. **Rewrite the inequality** as an equation to draw the boundary line.
2. **Choose the correct line style**:
   - **Solid line** for $\leqslant$ or $\geqslant$
   - **Dashed (broken) line** for $<$ or $>$
3. **Shade the region** that satisfies the inequality.
   - Test a point (e.g. (0,0)) to check if it satisfies the inequality.
   - If yes, shade the side that includes that point.

Example 4:

Graph:

$y < 2x + 1$

- Draw a dashed line for $y = 2x + 1$
- Shade **below** the line (since y is less than).

Example 5:

Graph:

$y \geqslant -x + 3$

- Draw a **solid** line for $y = -x + 3$
- Shade **above** the line.

4. List Inequalities that Define a Given Region

If a shaded region is shown on a graph, determine all the inequalities that define it.

Steps:

1. Identify the boundary lines and write their equations.
2. Determine if each line is dashed or solid.
3. Determine the direction of the inequality (above/below or left/right of the line).
4. Combine all the inequalities to define the region.

Example 6:

A region is bounded by:

- $x \geqslant 0$ (right of y-axis)
- $y \geqslant 0$ (above x-axis)
- $y \leqslant 3$ (below the horizontal line y = 3)
- $x + y \leqslant 5$ (below the line x + y = 5)

Inequalities:

- $x \geqslant 0$
- $y \geqslant 0$
- $y \leqslant 3$
- $x + y \leqslant 5$

This set of inequalities defines the **feasible region** (note: not a linear programming problem here, just identifying the region).