📘 Pythagoras' Theorem in 3D

In 3D geometry, we can apply Pythagoras' Theorem to find the lengths of line segments (diagonals, edges, etc.) across different planes.

🔷 General 3D Pythagoras Formula:

For a point at coordinates (x, y, z), the distance from the origin $(0, 0, 0)$ is:

$$\text{Length} = \sqrt{x^2 + y^2 + z^2}$$

✅ Example 1: Finding the length of a diagonal in a cuboid

A cuboid has dimensions $6\,\text{cm} \times 4\,\text{cm} \times 3\,\text{cm}$. Find the length of the space diagonal.

Solution:

$$\text{Diagonal} = \sqrt{6^2 + 4^2 + 3^2} = \sqrt{36 + 16 + 9} = \sqrt{61} \approx 7.81\,\text{cm}$$

📘 Trigonometry in 3D

You often need to:

- Use **right-angled triangles** in different planes.
- Break a 3D shape into **2D cross-sections** to apply sine, cosine, or tangent.

✅ Example 2: Finding an angle in 3D

In a cuboid, $AB = 5\,\text{cm}$, $BC = 4\,\text{cm}$, $BF = 3\,\text{cm}$. Find angle between diagonal AF and base AB.

- Triangle ABF is right-angled at B.
- Use coordinates or visual reasoning.

Let's find length AF using Pythagoras:

$$AF = \sqrt{AB^2 + BF^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$$

Now use **cosine rule** in triangle ABF, or simply:

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AF} = \frac{5}{\sqrt{34}} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{\sqrt{34}}\right)$$

📘 Angle Between a Line and a Plane

The **angle between a line and a plane** is the **complement** of the angle between the line and its projection on the plane.

🧠 **Key idea**:

$$\sin(\theta) = \frac{\text{component perpendicular to plane}}{\text{length of the line}}$$

Or using vector method: If **line vector** is $\vec{a}$, and **plane has normal vector** $\vec{n}$, then:

$$\sin(\theta) = \frac{|\vec{a} \cdot \vec{n}|}{|\vec{a}||\vec{n}|}$$

Then:

$$\theta = \sin^{-1}\left(\frac{|\vec{a} \cdot \vec{n}|}{|\vec{a}||\vec{n}|}\right)$$

✅ Example 3: Line–plane angle

Let vector of line be $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, and the plane has normal vector $\vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

Then:

- $\vec{a} \cdot \vec{n} = 2 \cdot 0 + 1 \cdot 0 + 3 \cdot 1 = 3$
- $|\vec{a}| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$, $|\vec{n}| = 1$

$$\sin(\theta) = \frac{3}{\sqrt{14}} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

💡 Tips for 3D Geometry Problems

- Sketch the 3D object.
- Break it into right-angled triangles.
- Use **Pythagoras** or **trig ratios** in the triangle.
- Use **vector methods** when dealing with direction or planes.