📘 Circle Theorems: Notes & Examples

1. Angle in a Semicircle = 90°

🔷 **Theorem:**

Any triangle drawn from the diameter of a circle creates a right angle (90°) at the circumference.

✅ **Explanation:**

If angle ∠ABC is subtended by a diameter AC, then ∠ABC = 90°.

🧮 **Example:**

- Given: AC is the diameter, and B lies on the circle.
- Find: ∠ABC
- Solution:
  ∠ABC = 90° (Angle in a semicircle)

2. Angle Between Tangent and Radius = 90°

🔷 **Theorem:**

A tangent to a circle is perpendicular to the radius at the point of contact.

✅ **Explanation:**

If O is the center, and a tangent touches the circle at point T, then ∠OTX = 90°, where OT is the radius and TX is the tangent.

🧮 **Example:**

- Given: OT is a radius and TX is a tangent at T.
- Find: ∠OTX
- Solution:
  ∠OTX = 90° (Angle between radius and tangent)

3. Angle at the Centre is Twice the Angle at the Circumference

🔷 Theorem:

The angle subtended by an arc at the centre is **twice** the angle subtended at the circumference.

✅ Explanation:

If ∠AOC is the angle at the center and ∠ABC is at the circumference, then ∠AOC = 2 × ∠ABC.

🧮 Example:

- Given: ∠ABC = 30°
- Find: ∠AOC
- Solution:
  ∠AOC = 2 × 30° = 60° (Angle at centre is twice angle at circumference)

4. Angles in the Same Segment are Equal

🔷 Theorem:

Angles subtended by the same arc (or chord) on the same side of the chord are equal.

✅ Explanation:

If points A, B, C, and D lie on a circle such that arcs AC and AD subtend angles ∠ABC and ∠ADC respectively, then ∠ABC = ∠ADC.

🧮 Example:

- Given: ∠ABC = 40°, and angle ∠ADC is subtended by the same arc.
- Find: ∠ADC
- Solution:
  ∠ADC = 40° (Angles in the same segment)

5. Opposite Angles of a Cyclic Quadrilateral Sum to 180°

🔷 Theorem:

In a **cyclic quadrilateral** (a quadrilateral inscribed in a circle), **opposite angles add up to 180°**.

✅ Explanation:

If ABCD is a cyclic quadrilateral, then:

- $\angle A + \angle C = 180°$
- $\angle B + \angle D = 180°$

🧮 Example:

- Given: $\angle A = 105°$
- Find: $\angle C$
- Solution:
  $\angle C = 180° - 105° = 75°$ (Opposite angles of cyclic quad)

6. Alternate Segment Theorem

🔷 Theorem:

The angle between a tangent and a chord is **equal** to the angle in the alternate segment.

✅ Explanation:

If a tangent touches the circle at point A and chord AB is drawn, the angle between the **tangent and chord AB** is equal to the angle in the opposite segment of the circle.

🧮 Example:

- Given: Angle between tangent and chord AB = 50°
- Find: Angle in alternate segment (e.g., $\angle ACB$)
- Solution:
  $\angle ACB = 50°$ (Alternate segment theorem)

📝 Tips for Solving Problems:

- **Label the diagram** clearly.
- Use a **ruler and compass** to construct accurate figures if needed.
- Write **reasons** for each angle using the correct theorem.
- Look for **diameters**, **cyclic quadrilaterals**, **tangents**, and **equal arcs**.

📘 Circle Theorems II – Notes and Examples

🔁 Symmetry Properties of Circles

1. **Equal Chords are Equidistant from the Centre**
   - **Meaning**: If two chords in a circle are the same length, they are the same distance from the centre of the circle.
   - **Visual Clue**: Chords look like mirror images across a line through the centre.
   - **Reason to Use in Proofs**: "Equal chords are equidistant from the centre."
   - **Example**:
     In a circle with chords AB and CD, if AB = CD, then the perpendicular distances from the centre to AB and CD are equal.
2. **The Perpendicular Bisector of a Chord Passes Through the Centre**
   - **Meaning**: If you draw a perpendicular line through the midpoint of any chord, it will go through the centre of the circle.
   - **Useful for**:
     - Locating the centre of a circle.
     - Proving points lie on the same circle (circle construction).
   - **Reason to Use in Proofs**: "The perpendicular bisector of a chord passes through the centre."
   - **Example**:
     In triangle ABC, if D is the midpoint of chord AB and line OD is perpendicular to AB, then line OD passes through the centre O.
3. **Tangents from an External Point are Equal in Length**
   - **Meaning**: If you draw two tangents from the same point outside a circle, the segments from that point to where they touch the circle are equal.
   - **Used to Prove**:
     - Triangles formed are congruent.
     - Tangent segments have equal lengths.
   - **Reason to Use in Proofs**: "Tangents from an external point are equal."
   - **Example**:
     From a point P outside a circle, tangents PA and PB touch the circle at A and B. Then, PA = PB.

✏️ Quick Practice Questions

1. In a circle, chords AB and CD are equal. Prove that the distances from the centre to AB and CD are equal.
   ➤ Use: "Equal chords are equidistant from the centre."
2. A chord AB is bisected by line OC, and OC is perpendicular to AB. What can you say about point O?
   ➤ Use: "The perpendicular bisector of a chord passes through the centre."
3. From an external point P, tangents PA and PB are drawn to a circle. Prove triangle PAB is isosceles.
   ➤ Use: "Tangents from an external point are equal."