🔷 1. Understand and Use Indices

This includes positive, zero, negative, and fractional indices.

🔶 What are Indices?

Indices (or exponents/powers) show how many times a number (called the base) is multiplied by itself.

Example:

$2^3 = 2 \times 2 \times 2 = 8$

✅ Positive Indices

A positive index tells you how many times to multiply the base by itself.

Example:

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

✅ Zero Index

Any non-zero number raised to the power of 0 is 1.

Rule:

$a^0 = 1$ (where $a \neq 0$)

Examples:

- $5^0 = 1$
- $(-2)^0 = 1$

✅ Negative Indices

A negative index means the reciprocal (1 over the number) of the base raised to the positive power.

Rule:

$a^{-n} = \frac{1}{a^n}$

Examples:

- $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
- $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

✅ Fractional Indices

A fractional index is related to roots.

Rules:

- $a^{\frac{1}{n}} = \sqrt[n]{a}$ (n-th root of a)
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Examples:

- $9^{\frac{1}{2}} = \sqrt{9} = 3$
- $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
- $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

🔷 2. Understand and Use the Rules of Indices

✅ Rule 1: Multiplying Powers with the Same Base

Rule:

$a^m \times a^n = a^{m+n}$

Example:

$2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$

✅ Rule 2: Dividing Powers with the Same Base

Rule:

$\frac{a^m}{a^n} = a^{m-n}$

Example:

$\frac{5^6}{5^2} = 5^{6-2} = 5^4 = 625$

✅ Rule 3: Power of a Power

Rule:

$(a^m)^n = a^{mn}$

Example:

$(3^2)^4 = 3^{2\times4} = 3^8 = 6561$

✅ Rule 4: **Power of a Product**

Rule:

$(ab)^n = a^n \times b^n$

Example:

$(2 \times 5)^3 = 2^3 \times 5^3 = 8 \times 125 = 1000$

✅ Rule 5: **Power of a Fraction**

Rule:

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example:

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

📝 Summary Chart of Index Rules:

Rule	Formula	Example
Multiply	$a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^7$
Divide	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{5^6}{5^2} = 5^4$
Power of Power	$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^8$
Zero Power	$a^0 = 1$	$7^0 = 1$
Negative Power	$a^{-n} = \frac{1}{a^n}$	$10^{-2} = \frac{1}{100}$
Fractional Power	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$27^{\frac{2}{3}} = 9$
Product Power	$(ab)^n = a^n \times b^n$	$(2 \cdot 3)^2 = 2^2 \cdot 3^2$
Fraction Power	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$