

# **IGCSE Mathematics: Sequences – Notes & Examples**



# 1. Continue a Given Number Sequence or Pattern



#### Rey Idea:

You must observe the pattern or rule that connects the terms of a sequence and use it to find the next term(s).

# Example 1:

Sequence: 2, 4, 6, 8, \_\_, \_\_

Pattern: Each term increases by 2 (add 2 each time).

Next terms: 10, 12

### Example 2:

Sequence: 1, 4, 9, 16, \_\_, \_\_

Pattern: These are square numbers:  $1^2, 2^2, 3^2, 4^2, \dots$ 

Next terms:  $5^2 = 25, 6^2 = 36$ 



## 2. Recognise Patterns in Sequences

# Key Concepts:

- Look for term-to-term rules: How to get from one term to the next.
- Recognise types of sequences: linear, quadratic, etc.
- Identify relationships between two sequences (e.g. one is double the other).

## Example 1 (Term-to-Term Rule):

Sequence: 10, 7, 4, 1, \_\_, \_\_

Rule: Subtract 3 Next terms: -2, -5

# Example 2 (Relationship Between Two Sequences):

A: 1, 2, 3, 4, 5

B: 2, 4, 6, 8, 10

Rule: B is double A  $\rightarrow B_n = 2A_n$ 

# 3. Find and Use the nth Term of Sequences

#### 

- Linear:  $T_n = an + b$
- Quadratic:  $T_n = an^2 + bn + c$
- Cubic:  $T_n = an^3 + bn^2 + cn + d$
- Exponential:  $T_n = a r^{n-1}$

# A. Linear Sequences

Increases/decreases by a constant amount.

### Method to find nth term:

- 1. Find the common difference (d)
- 2. nth term = dn + (firstterm d)

## Example:

Sequence: 5, 8, 11, 14, ...

Common difference: +3

$$T_n = 3n + (5-3) = 3n + 2$$

# B. Quadratic Sequences

The second difference is constant.

### Method:

- 1. Find first and second differences.
- 2. Assume nth term is  $an^2 + bn + c$
- 3. Form equations using the first few terms.
- 4. Solve to find a, b, and c.

## Example:

Sequence: 2, 6, 12, 20, 30, ...

1st differences: +4, +6, +8, +10

2nd differences: +2 → Quadratic

Assume  $T_n = an^2 + bn + c$ 

Use values:

• 
$$T_1 = a + b + c = 2$$

• 
$$T_2 = 4a + 2b + c = 6$$

• 
$$T_3 = 9a + 3b + c = 12$$

Solve the system to get:

$$a = 1, b = 0, c = 1$$

So, 
$$T_n=n^2+1$$

# C. Cubic Sequences

The third difference is constant.

## Example:

Sequence: 1, 8, 27, 64, ...

This is  $1^3, 2^3, 3^3, 4^3$ 

nth term:  $T_n=n^3$ 

# D. Exponential Sequences

Each term is multiplied by a fixed number.

## Example:

Sequence: 3, 6, 12, 24, 48, ...

Rule: Multiply by 2 each time

nth term:  $T_n = 3 \cdot 2^{n-1}$ 



### E. Mixed/Combination Sequences

Can combine linear and quadratic or exponential.



Sequence: 
$$T_n = n^2 + 2n$$

Find first 3 terms:

• 
$$T_1 = 1 + 2 = 3$$

• 
$$T_2 = 4 + 4 = 8$$

• 
$$T_3 = 9 + 6 = 15$$

# Tips for Solving Sequence Problems

- Always write down the first few terms.
- Check differences (first, second, third) to determine the type.
- Use subscript notation  $T_n$  to clearly show the nth term.
- Double-check using a few values of n.