📘 Proportion Notes (IGCSE Level)

🔷 1. Proportional Symbol (∝)

The symbol ∝ means "is proportional to".

If $y \propto x$, it means as x increases, y increases at a constant ratio (directly proportional).

🔷 2. Direct Proportion

- If $y \propto x$, then $y = kx$ for some constant k.
- This means: When one quantity increases, the other increases at the same rate.

✅ **Examples:**

- **Linear Direct Proportion:**
  $y \propto x \rightarrow y = kx$
  If $y = 10$ when $x = 2$, find y when $x = 5$.
  $\rightarrow k = \frac{y}{x} = \frac{10}{2} = 5$
  $\rightarrow y = 5 \times 5 = 25$
- **Square Proportion:**
  $y \propto x^2 \rightarrow y = kx^2$
- **Square Root Proportion:**
  $y \propto \sqrt{x} \rightarrow y = k\sqrt{x}$
- **Cube Proportion:**
  $y \propto x^3 \rightarrow y = kx^3$
- **Cube Root Proportion:**
  $y \propto \sqrt[3]{x} \rightarrow y = k\sqrt[3]{x}$

🔷 3. Inverse Proportion

- If $y \propto \frac{1}{x}$, then $y = \frac{k}{x}$
- This means: As one quantity increases, the other decreases.

✅ Examples:

- Linear Inverse Proportion:
  $y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$
- Square Inverse Proportion:
  $y \propto \frac{1}{x^2} \rightarrow y = \frac{k}{x^2}$
- Square Root Inverse Proportion:
  $y \propto \frac{1}{\sqrt{x}} \rightarrow y = \frac{k}{\sqrt{x}}$
- Cube Inverse Proportion:
  $y \propto \frac{1}{x^3} \rightarrow y = \frac{k}{x^3}$
- Cube Root Inverse Proportion:
  $y \propto \frac{1}{\sqrt[3]{x}} \rightarrow y = \frac{k}{\sqrt[3]{x}}$

🔷 4. Solving Proportion Problems

1. **Write the proportional relationship** using $\propto$
2. **Convert to equation** with a constant k
3. **Find** k using given values
4. **Use the equation** to find the unknown

🧠 Example Questions

Example 1 (Direct Square):

> $y \propto x^2$. Given $y = 36$ when $x = 3$, find y when $x = 5$.

Solution:

$$y = kx^2 \Rightarrow 36 = k \cdot 9 \Rightarrow k = 4$$
$$y = 4 \cdot 25 = 100$$

Example 2 (Inverse Cube Root):

> $y \propto \frac{1}{\sqrt[3]{x}}$. Given $y = 2$ when $x = 8$, find y when $x = 27$.

Solution:

$$y = \frac{k}{\sqrt[3]{x}} \Rightarrow 2 = \frac{k}{2} \Rightarrow k = 4$$
$$y = \frac{4}{\sqrt[3]{27}} = \frac{4}{3}$$