1. Linear Functions

General form:

$y = ax + b$ or rearranged from: $ax + by = c$

Key features:

- Straight line graph
- Gradient = a
- y-intercept = b

Example:

$y = 2x + 1$

- Gradient: 2 → Line rises 2 units for every 1 unit across
- y-intercept: 1 → Crosses y-axis at (0, 1)

Sketch steps:

1. Plot y-intercept: (0, 1)
2. Use gradient to find another point: e.g., (1, 3)
3. Draw a straight line

2. Quadratic Functions

General form:

$y = ax^2 + bx + c$

Key features:

- Parabola (U-shaped or ∩-shaped)
- **Turning point**: vertex (minimum if $a > 0$, maximum if $a < 0$)
- **Axis of symmetry**: vertical line through the turning point
- **Roots**: x-values where $y = 0$

Finding the turning point (by completing the square):

$y = ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$

Example:

$y = x^2 - 4x + 3$

Completing the square:

$y = (x - 2)^2 - 1$ → Turning point: (2, -1), Axis of symmetry: $x = 2$, Roots: Solve $x^2 - 4x + 3 = 0$ → $x = 1, 3$

Sketch steps:

1. Plot turning point
2. Plot roots (x-intercepts)
3. Plot y-intercept ($c = 3$)
4. Draw a smooth curve

---

3. Cubic Functions

General forms:

- $y = ax^3 + b$
- $y = ax^3 + bx^2 + cx + d$

Key features:

- May have **1 or 3 real roots**
- May have **a turning point or two turning points**
- Odd symmetry (if centered at origin or turning point)

Example:

$y = x^3 - 3x$

- Factor: $y = x(x^2 - 3)$ → Roots: $x = 0, \sqrt{3}, -\sqrt{3}$
- Turning points by differentiation or plotting

Sketch steps:

1. Find and plot roots
2. Estimate turning points (or find them using calculus if required)
3. Sketch with correct shape

4. Reciprocal Functions

General form:

$y = \frac{a}{x} + b$

Key features:

- **Two branches**
- **Vertical asymptote:** $x = 0$ (graph never touches x = 0)
- **Horizontal asymptote:** $y = b$
- Decreasing in one quadrant, increasing in the other

Example:

$y = \frac{2}{x}$

Sketch steps:

1. Mark asymptotes: x = 0, y = 0
2. Plot points for positive and negative x values
3. Sketch smooth hyperbola in opposite quadrants

5. Exponential Functions

General form:

$y = ar^x + b$, where $r > 0$

Key features:

- Curve rises (if $r > 1$) or decays (if $0 < r < 1$)
- **Horizontal asymptote:** $y = b$
- No x-intercepts if $b > 0$

Example:

$y = 2^x$

- Rapidly increases for positive x
- Approaches 0 for negative x

Sketch steps:

1. Plot basic values (e.g., $x = -2, -1, 0, 1, 2$)
2. Mark horizontal asymptote: $y = 0$
3. Sketch smooth curve

Summary Table

Function Type	Shape	Key Features
Linear	Straight line	Gradient and intercept
Quadratic	Parabola	Turning point, symmetry, roots
Cubic	S-curve	Up to 3 roots, turning points
Reciprocal	Two branches	Asymptotes, undefined at x = 0
Exponential	Rapid rise or decay	Horizontal asymptote, always positive