🌟 IGCSE Differentiation Notes with Examples

1. Estimate Gradients of Curves by Drawing Tangents

🔷 Concept:

The **gradient** of a curve at a point is the gradient of the **tangent line** at that point.

🔷 Steps to Estimate:

1. Place a **ruler** so it touches the curve at one point only (the point of interest).
2. Draw the **tangent line**.
3. Choose two points on the tangent line (not the curve).
4. Use the formula:

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

✅ Example:

A curve passes through point (2, 4). You draw a tangent at this point and choose (1, 2) and (3, 6) on the tangent line.

$$\text{Gradient} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

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2. Use Derivatives of Functions of the Form ax^n

🔷 Power Rule:

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

Where:

- a is a constant
- n is a positive integer or zero

✅ Examples:

1. $y = 5x^3 \Rightarrow \frac{dy}{dx} = 15x^2$
2. $y = 3x^2 + 2x + 7 \Rightarrow \frac{dy}{dx} = 6x + 2$
3. $y = 4x^4 - x^2 + 3 \Rightarrow \frac{dy}{dx} = 16x^3 - 2x$

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3. Apply Differentiation to Gradients and Stationary Points

🔷 **Gradient at a Point:**

Find the derivative, then substitute the x-value to get the gradient.

🔷 **Stationary Points:**

These occur when:

$$\frac{dy}{dx} = 0$$

To find the **coordinates** of the stationary point:

1. Differentiate.
2. Set derivative to 0 and solve for x.
3. Substitute back into the original function to find y.

✅ **Example:**

Given $y = x^2 - 4x + 3$

1. $\frac{dy}{dx} = 2x - 4$
2. Set $2x - 4 = 0 \Rightarrow x = 2$
3. $y = 2^2 - 4(2) + 3 = 4 - 8 + 3 = -1$

Stationary point is at (2, -1)

4. Discriminate Between Maxima and Minima

🔷 **Methods:**

(i) Sketching:

Draw the curve to see whether the turning point is a **maximum** (peak) or **minimum** (valley).

(ii) Second Derivative Test:

Find the second derivative $\frac{d^2y}{dx^2}$

- If $\frac{d^2y}{dx^2} > 0$, it's a **minimum point**
- If $\frac{d^2y}{dx^2} < 0$, it's a **maximum point**

(iii) Gradient Either Side:

Choose values **just before and just after** the stationary point and check the gradient:

- Gradient changes from **positive to negative → Maximum**
- Gradient changes from **negative to positive → Minimum**

✅ **Example:**

Given $y = x^2 - 4x + 3$

1. First derivative: $\frac{dy}{dx} = 2x - 4$
2. Stationary point: $x = 2$
3. Second derivative: $\frac{d^2y}{dx^2} = 2$ → **Positive**, so it's a **minimum**

📘 Summary Table

Function	Derivative	Stationary Point	Nature
$y = x^2 - 4x + 3$	$2x - 4$	$x = 2, y = -1$	Minimum
$y = -x^2 + 4x - 3$	$-2x + 4$	$x = 2, y = 1$	Maximum