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IGCSE Differentiation Notes with Examples

1. Estimate Gradients of Curves by Drawing Tangents

♦ Concept:

The gradient of a curve at a point is the gradient of the tangent line at that point.

Steps to Estimate:

- 1. Place a ruler so it touches the curve at one point only (the point of interest).
- 2. Draw the tangent line.
- 3. Choose two points on the tangent line (not the curve).
- 4. Use the formula:

$$ext{Gradient} = rac{ ext{Change in } y}{ ext{Change in } x} = rac{y_2 - y_1}{x_2 - x_1}$$

Example:

A curve passes through point (2, 4). You draw a tangent at this point and choose (1, 2) and (3, 6) on the tangent line.

Gradient =
$$\frac{6-2}{3-1} = \frac{4}{2} = 2$$

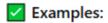
2. Use Derivatives of Functions of the Form ax^n

♦ Power Rule:

$$rac{d}{dx}(ax^n) = anx^{n-1}$$

Where:

- a is a constant
- n is a positive integer or zero



1.
$$y=5x^3\Rightarrow \frac{dy}{dx}=15x^2$$

2.
$$y = 3x^2 + 2x + 7 \Rightarrow \frac{dy}{dx} = 6x + 2$$

3.
$$y = 4x^4 - x^2 + 3 \Rightarrow \frac{dy}{dx} = 16x^3 - 2x$$

3. Apply Differentiation to Gradients and Stationary Points

Gradient at a Point:

Find the derivative, then substitute the x-value to get the gradient.

♦ Stationary Points:

These occur when:

$$\frac{dy}{dx} = 0$$

To find the coordinates of the stationary point:

- 1. Differentiate.
- 2. Set derivative to 0 and solve for x.
- **3.** Substitute back into the original function to find y.

Example:

Given
$$y=x^2-4x+3$$

1.
$$\frac{dy}{dx} = 2x - 4$$

2. Set
$$2x - 4 = 0 \Rightarrow x = 2$$

3.
$$y = 2^2 - 4(2) + 3 = 4 - 8 + 3 = -1$$

Stationary point is at (2, -1)

4. Discriminate Between Maxima and Minima

Methods:

(i) Sketching:

Draw the curve to see whether the turning point is a maximum (peak) or minimum (valley).

(ii) Second Derivative Test:

Find the second derivative $\frac{d^2y}{dx^2}$

- If $rac{d^2y}{dx^2}>0$, it's a minimum point
- If $\frac{d^2y}{dx^2} < 0$, it's a maximum point

(iii) Gradient Either Side:

Choose values just before and just after the stationary point and check the gradient:

- Gradient changes from positive to negative → Maximum
- Gradient changes from negative to positive → Minimum

Example:

Given
$$y=x^2-4x+3$$

- 1. First derivative: $\frac{dy}{dx}=2x-4$
- 2. Stationary point: x=2
- 3. Second derivative: $\frac{d^2y}{dx^2}=2$ \rightarrow Positive, so it's a minimum

Summary Table

Function	Derivative	Stationary Point	Nature
$y=x^2-4x+3$	2x-4	x=2, $y=-1$	Minimum
$y=-x^2+4x-3$	-2x+4	x=2, $y=1$	Maximum