1. Understanding Functions, Domain and Range, and Function Notation

Function Notation

- A function is written as:
  $f(x)$, which means "the function of x"
  Example:
  $f(x) = 2x + 1$
- You can substitute values:
  If $f(x) = 2x + 1$, then
  $f(3) = 2(3) + 1 = 7$

Domain

- The **domain** is the set of all **input values** (x-values) that a function can take.
  - Example: If $f(x) = \frac{1}{x}$, the domain is all real numbers except $x \neq 0$

Range

- The **range** is the set of all **output values** (y-values) the function can produce.
  - Example: If $f(x) = x^2$, and domain is $x \in \mathbb{R}$, then range is $y \geq 0$

Mapping Diagrams

- Visual way to show inputs and outputs:
  - Example:
    Inputs: 1, 2, 3
    Rule: $f(x) = 2x$
    Outputs: 2, 4, 6

2. Inverse Functions $f^{-1}(x)$

Definition

- An **inverse function** reverses the original function.
- If $f(x) = y$, then $f^{-1}(y) = x$

Finding the Inverse

Steps:

1. Let $y = f(x)$
2. Swap x and y
3. Solve for y

Example: Let $f(x) = 3x - 5$

1. $y = 3x - 5$
2. $x = 3y - 5$
3. $x + 5 = 3y$
4. $y = \frac{x+5}{3}$
   So, $f^{-1}(x) = \frac{x+5}{3}$

3. Composite Functions (e.g. $gf(x) = g(f(x))$)

- Apply one function and then another.
- $fg(x)$ means apply **g first**, then **f**

Examples

Given:

- $f(x) = 3x - 5$
- $g(x) = 3(x + 4)$
- $h(x) = 2x^2 + 3$

Example 1:

Find $gf(x)$
First: $f(x) = 3x - 5$
Now: $g(f(x)) = 3((3x - 5) + 4) = 3(3x - 1) = 9x - 3$
Answer: $gf(x) = 9x - 3$

Example 2:

Find $fg(x)$
Given:

- $f(x) = 3x + 2$
- $g(x) = (3x + 5)^2$

So, $fg(x) = f(g(x)) = 3[(3x + 5)^2] + 2$
Answer: $fg(x) = 3(3x + 5)^2 + 2$